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MHD formulations for the liquid metal flow in a curved pipe of circular cross section

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ABSTRACT

The laminar fully developed magnetohydrodynamic (MHD) flow of a liquid metal into a curved pipe of circular cross section, subjected to a transverse external magnetic field, is studied. Three different formulations are used for the implementation of the electromagnetic variables. The extended Continuity Vorticity Pressure (CVP) numerical variational method for MHD flows is used for the coupling of the momentum and the continuity equation. Results are obtained for different values of the curvature (0–0.2) and of the Hartmann number (0–1000). The magnitude of the axial velocity is determined by the balance of the centrifugal and the electromagnetic forces. The results reveal the limits of applicability of the used electromagnetic models as the Hartmann number increases.

1. Introduction

In magnetohydrodynamics (MHD), the motion of the electrically conducting fluids is studied under the effect of strong magnetic fields and involves many applications as internal, unbounded, free surface and ferrofluid flows. MHD liquid metal flows, subjected to strong external magnetic fields, are met in many practical applications related to fusion reactors, electromagnetic pumping, power generation and other engineering applications. Various designs of these applications involve curved ducts. The content of the present paper is focused on the MHD flow of a liquid metal within a curved circular pipe, as part of a cooling system for fusion reactor blankets for moderate values of curvature.

The solution of the Maxwell equations on a toroidal–poloidal geometry on MHD flows has been widely studied on astrophysical or plasma MHD problems (e.g. [1]), but only few theoretical and experimental research works have been published up to today on laboratory/industrial scale MHD channel flows on curved ducts (highly viscous and low magnetic Reynolds flows). Kobayashi [2] studied the effect of a perpendicular external magnetic field on the secondary vortex flow of a curved channel, for small Hartmann numbers up to 20, showing that the primary flow is stabilized by the magnetic field effect. Sudou et al. [3] performed a theoretical and experimental analysis for the flow of a liquid metal in a curved circular channel for small Hartmann numbers up to 20 and small curvatures. Their results show that as the magnetic field increases, the secondary flow is suppressed by the magnetic field and the velocity profile comes up to that in straight channels. The laminar MHD flow in a rectangular curved channel is studied by Tabeling and Chabrerie [4] at intermediate Hartmann numbers, using an approximate analytical perturbation method. Issacci et al. [5] studied the MHD flow in a circular channel using an approximate analytical method for small and intermediate Hartmann numbers and small curvatures. Moresco and Alboussière [6] with an experiment, and Vantieghem and Knaepen [7] with numerical simulation, studied the MHD flow on a closed toroidal loop of a square curved duct driven by an electric current at laminar and turbulent conditions. More specific, they studied the instability conditions for the Hartmann layer for small and intermediate Hartmann numbers ($Ha \leqslant 400$).

On the other hand, a great number of numerical studies for the fully developed MHD flows on straight ducts have been carried out in conducting or insulated channels within the finite-difference or the finite-element methods, e.g. [8–14] or the finite volume method on a collocated grid [15]. These methods differ in using two basic electromagnetic models, the electrical potential formulation (ϕ-formulation) or the induced magnetic field formulation (b-formulation), implementing different boundary conditions.

Hatzikonstantinou and Bakalis [16], investigating numerically the MHD flow in a straight insulated circular annular duct,

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determined the limits of applications of the low magnetic Reynolds number electrical field formulation (ϕ-formulation), the hybrid h-formulation and the induced magnetic field formulation (b-formulation). It was found that the b-formulation has some advantages for moderate values of $Ha \leqslant 500$, due to the accurate estimation of the transverse components of the induced magnetic field and of the transverse velocity components. However the simplified h-formulation yields to satisfactory results for $Ha \geqslant 500$, due to the fact that the transverse components of the induced magnetic field and of the velocity field are reduced dramatically as the Hartmann number increases to very high values.

The forced laminar fully developed magnetohydrodynamic (MHD) flow of a liquid metal moving in a curved circular channel, under the action of the axial pressure gradient and the effect of an external transverse magnetic field, is studied numerically at the present paper for the first time. This configuration could be a section of a bend or a turn of the fusion reactor blanket access ducts piping system [17,18]. The electromagnetic variables will be implemented with three different formulations. Results are presented for the curvatures $\kappa = 0.0, 0.05, 0.1, 0.2$ and Hartmann numbers in the range of $0 \leqslant Ha \leqslant 1000$. The computational method that was used to solve the Navier–Stokes and continuity equations was the Continuity Vorticity Pressure (CVP) method, which has been developed by Papadopoulos and Hatzikonstantinou and is analytically presented for 2D, quasi 3D and fully 3D flow patterns in [18]. The method has already been validated and tested in MHD channel flows in straight [16,20] and curved channels [21].

2. Problem formulation

We consider a curved duct of circular cross section, of radius R and of radius of curvature R_c (see Fig. 1a), as part of a cooling system structure. An incompressible electrically conducting liquid metal flows in the pipe, under the effect of an external vertical and upward directed transverse magnetic field $\vec{B}_o$, which is applied for the confinement of the plasma inside the fusion reactor. The walls are assumed to be electrically insulated.

The MHD flow is governed by the following set of non-dimensional equations:

Continuity equation

$$\vec{\nabla} \cdot \vec{V} = 0 \tag{1}$$

Momentum equation

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \vec{\nabla}\right)\vec{V} = -\vec{\nabla}P + \frac{1}{Re}\nabla^2\vec{V} + \frac{Ha^2}{Re}(\vec{J} \times \vec{B}) \tag{2}$$

Magnetic induction equation

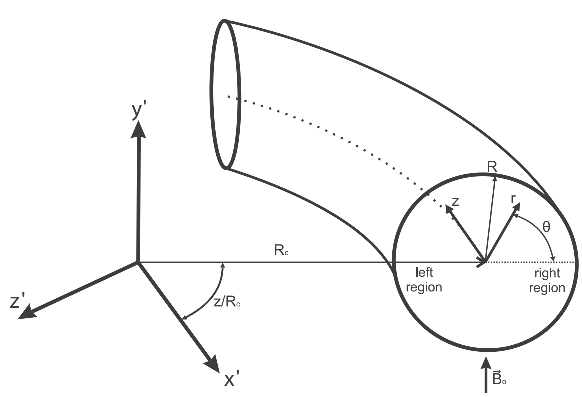

Fig. 1a. Toroidal–Poloidal coordinate system.

$$\frac{\partial \vec{B}}{\partial t} + \left(\vec{V} \cdot \vec{\nabla}\right)\vec{B} = \frac{1}{R_m}\nabla^2\vec{B} + \left(\vec{B} \cdot \vec{\nabla}\right)\vec{V} \tag{3}$$

Divergence free equation for the magnetic field

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{4}$$

Ohm's law

$$\vec{J} = -\vec{\nabla}\Phi + \vec{V} \times \vec{B} \tag{5}$$

Ampère's law

$$\vec{J} = \frac{1}{R_m}\vec{\nabla} \times \vec{B} \tag{6}$$

Divergence free equation for the electric current density

$$\vec{\nabla} \cdot \vec{J} = 0 \tag{7}$$

The use of the relations (5) and (6) depends on the electromagnetic formulation which is going to be used.

In the aforementioned Eqs. (1)–(7), $\vec{V}$ is the velocity, P is the pressure, $\vec{B} = \vec{B}_o + \vec{B}_i$ is the total magnetic field, given by the summation of the external $\vec{B}_0$ and the induced $\vec{B}_i$ magnetic field, Φ is the electric potential and $\vec{J}$ is the electric current density.

The toroidal–poloidal coordinate system (r, θ, z) of Fig. 1a is considered, which is connected to a Cartesian reference system (x', y', z') via the transformations $x' = (R_c + r\cos\theta)\cos(z/R_c)$, $y' = r\sin\theta$ and $z' = (R_c + r\cos\theta)\sin(z/R_c)$, where $0 \leqslant r \leqslant R$, $0 \leqslant \theta < 2\pi$ and $0 \leqslant z \leqslant 2\pi R_c$.

Eqs. (1)–(7) were made dimensionless using the scales $r = \frac{r'}{R'}$, $t = \frac{t'V'_0}{R'}$, $z = \frac{z'}{R'}$, $R_c = \frac{R'_c}{R'}$, $\vec{V} = \frac{\vec{V}'}{V'_0}$, $P = \frac{P'}{\rho V_0'^2}$, $\vec{B} = \frac{\vec{B}'}{B'_o}$, $\vec{J} = \frac{\vec{J}'}{\sigma V'_0 B'_o}$ and the following parameters: the Reynolds number $Re = V'_o R'/\nu$, the Hartmann number $Ha = \sqrt{\sigma/\rho\nu}B'_o R'$ and the magnetic Reynolds number $R_m = \mu\sigma V'_o R'$, where the prime "′" denotes the dimensional variables. On the above scales, R' is the dimensional radius of the circular cross section of the duct, ρ is the fluid density, ν is the kinematic viscosity of the fluid, V'_0 is the reference velocity, which is equal to the magnitude of the dimensional axial velocity, σ is the electric conductivity and μ is the magnetic permeability of the liquid metal, B'_o is the reference magnetic field, which is equal to the magnitude of the dimensional external magnetic field $\vec{B}'_o = B'_o\hat{y}$, where $\hat{y}$ is the unit normal vector in the Cartesian y'-direction.

In the internal fully developed forced laminar flows all variables $\vec{V} = u\hat{e}_r + v\hat{e}_\theta + w\hat{e}_z$, $\vec{B}$ and $\vec{J}$ are independent of the axial coordinate z, except for the pressure. Hence all the axial derivatives are neglected except for the axial pressure gradient $p_{a,z} \equiv \partial p_a(z)/\partial z$, which is regarded as uniform over the cross section and is updated during the iterative procedure by the mass conservation equation, so that the mean value of the axial velocity will be equal to $\bar{w} = 1$ [16,19,22]. The flow is subjected to a non-dimensional external constant magnetic field $\vec{B}_o = (B_{or}, B_{o\theta}, 0)$, where $B_{or} = \sin\theta, B_{o\theta} = \cos\theta$. The induced axial magnetic field is $\vec{B}_i = B_{ir}\hat{e}_r + B_{i\theta}\hat{e}_\theta + B_{iz}\hat{e}_z$, so that $\vec{B} = \vec{B}_o + \vec{B}_i = B_r\hat{e}_r + B_\theta\hat{e}_\theta + B_z\hat{e}_z$, and the produced total electric current density is $\vec{J} = J_r\hat{e}_r + J_\theta\hat{e}_\theta + J_z\hat{e}_z$. Here, we define $\hat{e}_r, \hat{e}_\theta, \hat{e}_z$ as the orthonormal basis in toroidal–poloidal coordinates. Also, it occurs that $\vec{\nabla} \times \vec{B}_o = \vec{0}$, because the external magnetic field is constant and it is generated by a non conducting media and, consequently, $\nabla^2\vec{B}_o = \vec{0}$, since $\vec{\nabla} \times \vec{\nabla} \times \vec{B}_o = \vec{\nabla}\vec{\nabla} \cdot \vec{B}_o - \nabla^2\vec{B}_o$ $(\vec{\nabla} \cdot \vec{B}_o = 0)$.

Hence, introducing the term $I = 1/(1 + \kappa r\cos\theta)$, where $\kappa = 1/R_c$ is the curvature, the governing equations of the MHD flow take the following form:

Continuity equation