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Extreme wave elevations beneath offshore platforms, second order trapping, and the near flat form of the quadratic transfer functions

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ABSTRACT

Extreme free surface elevations due to wave-structure interactions are investigated to second order using Quadratic Transfer Functions (QTFs). The near-trapping phenomenon for small arrays of closely spaced columns is studied for offshore applications, and the excitation of modes by linear and second order interactions is compared. A simple method for approximating near-trapped mode shapes is shown to give good results for both linear and second order excitation. Low frequency near-trapped mode shapes are shown to be very similar whether excited linearly or to second order. Approximating surface elevation sum QTF matrices as being flat perpendicular to the leading diagonal is investigated as a method for greatly reducing lengthy QTF calculations. The effect of this approximation on second order surface elevation calculations is assessed and shown to be reasonably small with realistic geometries for semi-submersible and tension-leg platforms.

1. Introduction

The phenomenon of near-trapping is a near-resonant local response excited by free-surface waves of a certain frequency interacting with arrays of obstacles such as vertical surface-piercing columns (see [14,4,5]) or with other geometries, including single bodies. Each near-trapping frequency is associated with a mode of strong local free surface oscillation which decays rather slowly in time due to wave radiation to infinity. However, the excitation periods of all but the lowest one or two near-trapped modes are usually too short to be significantly excited linearly by typical storm waves for most multi-column bodies as large as semi-submersible or tension-leg platforms. Non-linear wave responses can arise from various effects, such as the velocity squared term in the Bernoulli equation for pressure, and other non-linearities in the free surface boundary condition. The lowest order non-linear force is at sum and difference combination frequencies of the component incident wave frequencies. Second order sum frequency excitation of the higher near-trapped modes by waves with an incident period twice as long as the mode excitation period can form a large component of extreme wave-structure interactions [21,9]. Since second order responses can cause such a large contribution to the overall surface elevation, linear calculations are not sufficient to accurately model extreme wave-structure interactions. One must include second order contributions, despite the large increase in computational complexity, and the use of quadratic transfer functions (QTFs) is one possible method of modelling the second order responses in real sea-states. Quadratic transfer functions (defined below) are convolved with the incident surface elevation spectrum to give the response surface elevation spectrum, using the standard Volterra series approach described, for example, by Schetzen [18].

1.1. Transfer functions

Potential flow theory is used here to describe the incident waves, and the wave scattering by the structure. The unknown velocity potential, satisfying a non-linear boundary condition, is expressed as a perturbation expression in wave steepness, truncated at the second order terms (i.e. terms quadratic in wave amplitude). Eq. (1) describes the linear response elevation $\eta_R^{(1)}$ to two incident waves with amplitude A_i m, angular frequency ω_i rad/s, and phase ψ_i rad, where $i = 1, 2$ and b_i is the linear transfer function (LTF) at frequency ω_i. The second order response components are then given by Eq. (2) with the QTFs for the potential sum term b_{PS}, quadratic sum term b_{QS}, potential difference term b_{PD}, and quadratic difference term b_{QD}. This decomposition into quadratic and potential terms has been widely used by others to facilitate

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interpretation and verification of computed results (see, for example, early examples in [11,1]). The quadratic terms refer to the simple local product of two first order incident wave components. Potential terms arise from the inhomogeneous equations for the fluid velocity potential at second order and are driven by the interactions between pairs of incident frequency components. These are associated with the generation and propagation of free waves out to infinity as well as local contributions close to the structure. Sum terms refer to response at a frequency equal to the sum of the incident frequencies, $\omega_R = \omega_i + \omega_j$, (i.e. double the incident frequency for the self-interaction) and the difference terms refer to a response at $\omega_R = \omega_i - \omega_j$.

$$\eta_R^{(1)} = b_1\eta_1 + b_2\eta_2 = \Re\{b_1A_1e^{-i(\omega_1t+\psi_1)} + b_2A_2e^{-i(\omega_2t+\psi_2)}\} \tag{1}$$

$$\begin{aligned}\eta_R^{(2)} &= \Re\{(b_{PS} + b_{QS})A_1A_2e^{-i((\omega_1+\omega_2)t+\psi_1+\psi_2)}\} \\ &+ \Re\{(b_{PD} + b_{QD})A_1A_2e^{-i((\omega_1-\omega_2)t+\psi_1-\psi_2)}\}\end{aligned} \tag{2}$$

Here $\Re$ indicates that the real part is taken.

The quadratic transfer functions (QTFs) can be found using boundary element potential flow codes such as WAMIT (see [12,17]) or the Oxford code DIFFRACT (see [2,22,3]). These lead to the total surface elevation to second order in the vicinity of a structure for a given incident wave. Calculation of QTFs can be very computationally intensive and so it would be beneficial if a reasonable approximation could be found which reduced the number of QTF calculations necessary. Linear calculations are quick and cheap but have been shown to be insufficient when modelling extreme wave structure interactions, see for example Walker et al. [21], and Stansberg [19]. Calculation of each QTF not only takes much longer than for LTFs, but for an incident wave surface elevation spectrum with N frequency components one needs to fill four $N \times N$ matrices to cover the second order interactions between all possible pairs of frequency components. Use of symmetry when populating each matrix of QTFs can be used to reduce the number of calculations from N^2 to $N(N+1)/2$ (the leading diagonal plus one side) but this is still computationally expensive.

Taylor et al. [20] introduced a near-flat sum QTF matrix approximation for surface elevation around cylinder arrays. The authors observed that at low frequencies the sum QTF is a strong function of the output frequency ($\omega_S = \omega_i + \omega_j$) and virtually independent of the frequency difference ($\omega_D = \omega_i - \omega_j$), which is the distance away from the leading diagonal. This observation means that the whole QTF matrix might be approximated using only the leading diagonal. It would allow a reduction of the number of QTF calculations from $N(N+1)/2$ to N. This observation has the same empirical form as the Newman [16] approximation for difference frequency forces in vessels in irregular waves but Taylor et al. showed that a similar form of approximation is possible in second order sum surface elevation QTFs for arrays of cylinders. This approximation is investigated further to assess whether it is reasonable for use in wave-structure interaction analysis for certain types of configuration such as semi-submersible and tension leg platforms.

2. Near-trapped modes

Before beginning the lengthy process of calculating quadratic surface elevation transfer functions it is important to investigate the incident frequencies most likely to give a violent response. By finding the near-trapped mode frequencies for a given structure one can then plan the frequencies at which transfer functions should be calculated to give a reasonable model of extreme wave-structure interactions. The structure under study here is a simplified version of a typical large offshore platform. It consists of four vertical bottom-seated circular columns of radius a = 12.34 m, in water of depth 30 m, and with centres located at (±41.42 m, ±41.42 m). Fig. 1(a) shows the mesh for this simplified four circular column model and Fig. 1(b) shows the boundary mesh for a more realistic offshore structure. Only one quadrant is shown, as two planes of symmetry are assumed to minimise computation time. Analysis with the mesh in Fig. 1(b) will be discussed later. To identify the near-trapped frequencies for the simplified structure in Fig. 1(a) the method of Linton and Evans [13] was used, leading to thirteen near-trapped modes with a wavenumber less than 0.3 m^{-1}. Open ocean wave components with wavenumbers greater than this would have minimal energy and are therefore not considered.

It is also possible to identify complex wavenumbers at which theoretically a phenomenon of pure trapping occurs (with no radiation of waves away from the body). The method of Linton and Evans [13] makes use of a truncated infinite Fourier–Bessel series to model the total wave field including wave-structure interactions. When the matrix of coefficients associated with this truncated infinite series has a value of the determinant close to zero, a particularly violent response can occur. If a wavenumber leading to a zero in this determinant is real then pure-trapping has occurred. However, this only occurs for particular special geometries. In contrast, a much wider range of geometries leads to the phenomenon of near-trapped modes. The wavenumbers leading to zeros in the determinant are often complex and near-trapping may be thought of as the situation closest to pure-trapping if one sets the imaginary part of these complex wavenumbers to zero. The size of the imaginary part gives a measure of the wave damping due to radiation to infinity. The modes with the smallest imaginary wavenumber components are closest to pure-trapping with rather weak radiation leaking out to infinity and are therefore likely to have very large responses when excited by incoming waves. Detailed discussion on the linear excitation of these near-trapped modes for the same structure as considered here is given in Section 3 of Grice et al. [8]. A list of the predicted complex trapped mode wavenumbers is given in Table 1, which shows the real and imaginary parts of the wavenumber, normalised by column radius, and the associated period and wavelength for the simple four bottom-seated circular columns described above.

The first and lowest mode predicted has a normalised wavenumber of Re (ka) = 0.324 which for the geometry described above corresponds to an excitation period of 12.38 s. Typical storm waves on the open ocean have a peak period in the range $T_p = 12 - 15$ s. This means that the lowest few modes could be significantly excited linearly. For the higher modes the sea-state would have too little spectral energy for near-trapping to be excited through linear excitation. From a practical point of view, having found the near-trapped mode frequencies at which violent wave-structure interactions are most likely to occur, it is then useful to investigate the mode shapes, as this may lead to identification of the locations within the array where water-deck impact is most likely to occur.

2.1. Mode shape approximation

A method of approximating the shape of the free surface (termed the response) for a near-trapped mode is presented here. Using the method of Linton and Evans [13] to predict the near-trapped mode frequencies for arrays of cylinders, the associated mode shapes can be obtained based on series expansions. For more general multi-column configurations such as semi-submersibles, an alternative approach is desirable. The near-trapping frequencies may be obtained by observing peaks in the plots of characteristic parameters such as forces or local wave