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# A new flux-based scheme for compressible flows

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ABSTRACT

A new algorithm for the computing of compressible flows governed by Euler/Navier–Stokes equations is presented in this paper. The inter-cell numerical convective flux is estimated through a weighted combination of fourth order central/third order upwind biased/first order upwind interpolations of inter-cell numerical fluid velocity and convective transport vector. The higher order/lower order interpolations are carefully combined via two types of local solution sensitive weight functions. One of the weight functions is designed to control the balance of upwind/central contributions via flow speeds while the other one performs the dual purpose of detecting non-smooth or discontinuous features in the solution and regulating the balance between the higher order and first order upwind interpolations. The present work, through several one-dimensional (scalar and vector hyperbolic conservation laws) and multi-dimensional (Euler/Navier–Stokes) test cases, demonstrates that a carefully designed flux-based scheme can deliver a comparable performance in terms of robustness, accuracy and efficiency and is much simple to implement in comparison to some of the popular wave based TVD schemes like Van Leer and AUSMPW+ of Flux-Vector Splitting type and HLL scheme of Reconstruction-Evolution (Godunov) type. Employing multi-dimensional test cases, it is shown that, the new scheme is very robust and can be utilized for computing flows over a very wide range of flow speeds, ranging from incompressible limit (Mach No. ∼0.1) to very high speed compressible flows (Mach No. ∼10).

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## 1. Introduction

The presence of different types of discontinuities (shock waves, contacts, slip lines) together with their interactions with boundary/shear layers, vortices etc, poses a stiff challenge to the computing of compressible flows governed by Euler/Navier–Stokes equations. The requirements for resolving the discontinuities without any spurious oscillations or overshoots/undershoots and for resolving the flow structures like boundary layers, vortices etc are somewhat conflicting. The numerical scheme must possess sufficient numerical diffusion to stably resolve the discontinuities and yet not be overly diffusive to contaminate the viscous diffusion inside boundary/shear layers and vortices etc. In the light of the above, the design of a numerical scheme for the computing of compressible flows is a challenging and formidable task. The physics of compressible flows is dominated by propagation and interaction of waves of several families (entropy, vorticity and acoustic family). Therefore, it is generally believed that schemes that rely on the wave dynamics would capture the flow physics of compressible flows much better. This explains an almost one-sided effort in the development of numerical schemes for the computing of compressible flows. While much effort has been directed towards development of schemes that utilize the wave dynamics in a compressible flow, there has been little effort in the development of schemes that simply rely on direct discretization of the conservation equations aiming to conserve the fluxes on a numerical grid. The present work represents an effort in this direction.

### 1.1. Background and governing equations

The governing equations of an unsteady compressible flow exhibit a Hyperbolic character and thus any scheme that aims to resolve such a flow is built with the aid of one-dimensional model scalar/vector hyperbolic equations like Linear Advection Equation and Inviscid Burger's Equation, herein after referred to as LAE and IBE respectively, and canonical flows governed by Euler Equations. The inviscid model problems play a special role in highlighting the resolving capabilities of any numerical scheme as far as discontinuities are concerned. This is because, in the absence of viscosity, the discontinuities appear as true discontinuities in the solutions of the inviscid flow models and thus can only be stably resolved through numerical diffusion/artificial viscosity of the scheme. The numerical solution of the inviscid flow models,

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therefore, clearly identify the effects of numerical diffusion/artificial viscosity and allows for limiting these effects in a numerical scheme in order to achieve a desired resolving capability.

The numerical schemes for models represented by Hyperbolic Conservation Laws are constructed using two distinct approaches:

(1) coupled space–time approach and,
(2) decoupled space–time approach.

In a typical coupled approach, the discretization errors are estimated using the governing equation(s) themselves to achieve a high order of accuracy in the numerical scheme. In other words, the total truncation error, in time as well as in space is controlled. A typical example is a one step, Lax-Wendroff family of schemes. In contrast, in a decoupled space–time approach, the time integration is performed using either first order Euler or a two-step predictor–corrector or a multi-step RK method. The spatial discretization of the flux in this class of methods is achieved via three distinct approaches: (i) direct discretization using forward, backward or central schemes or their combinations (**flux-based approach**), (ii) flux-splitting followed by discretization (**wave based approach**), (iii) solution reconstruction using piecewise interpolation followed by solution of a cell based Riemann problem to estimate the intercell flux (**Reconstruction-evolution or Godunov approach**).

While the coupled space–time approach permits achieving very high orders of accuracy for the 1D scalar hyperbolic conservation laws, it is generally not possible to employ the Lax-Wendroff approach to develop schemes having more than second order accuracy for systems of hyperbolic equations representing multi-dimensional scenarios [1]. A common feature in all the methods (coupled or decoupled) is the presence of a shock/discontinuity detection algorithm that permits to limit the flux in order to minimize and control the oscillations/overshoots/undershoots in the solution in the vicinity of a discontinuity.

In the past, much work has been done in the development of shock capturing schemes employing the *decoupled space–time* approach. Among these methods, the **flux based methods** are simplest and easiest to implement. In these methods, the wave dynamics is not exploited to estimate the fluxes and their derivatives. Rather these are obtained by the direct discretization using forward, backward or central schemes on a computational grid. The most popular of such class of methods are Richtmyer and MacCormack's method [2]. These methods suffer from stability and accuracy in the presence of strong shock. In fact, without the addition of explicit artificial viscosity or damping terms, these methods produce severe oscillations/overshoots or undershoot in the vicinity of shocks.

The **wave based methods** carry out the spatial discretization of the governing equations by taking into consideration the direction and speed of the propagating family of waves [2–4]. The first-order upwind methods require large enough artificial viscosity for oscillation free solution in the vicinity of shocks. Due to this they experience severe smearing of contact discontinuities and shocks leading to reduced accuracy in smooth regions of the flow. Higher-order upwind methods, yield accurate solution in the smooth region but require explicit use of artificial viscosity or flux-limiters to prevent spurious oscillations/overshoots/under shoots in the vicinity of shocks or steep gradients in the solution. Another feature associated with most Flux-Vector Splitting (FVS) type methods is the difficulty experienced at the *sonic points*.

Another class of decoupled space–time wave based schemes are the Reconstruction-Evolution methods. As the name suggests, these methods approximate or reconstruct the solution at a given time instant by using piecewise polynomial approximations over each cell. The polynomial approximation is done in a manner that yields jump discontinuities at the cell interfaces. In order to evolve the solution in time, the discontinuity across the cell interface is regarded as localized Riemann problem between the two nodes across the cell interface. The Reconstruction-Evolution methods have excellent resolution capabilities of shocks and contacts discontinuities as they build up the solution from the solution of the localized Riemann problems. However, higher order methods require the use of limiters to stabilize the solution in the neighborhood of steep gradients. The main limitation of these classes of methods is large computational cost associated with the solution of large number of Riemann problems at each time step.

Another class of methods are essentially solution-sensitive TVD methods. These are methods that try to blend the basic methods of one family (wave-based or flux-based) described above where the exact amount of blending varies from place to place based on solution features such as shocks. The basic idea is to achieve best results both in smooth regions and in the regions of large gradients. The works of Van-Leer [5], Sweby [6] and Chakravarthy-Osher [7] can be regarded as major pioneering efforts in the development of TVD methods. These schemes vary in their complexity and level of sophistication to meet the conflicting requirement for the resolution of the discontinuities as well as the smooth flow features.

Much effort has been directed at development of accurate and robust wave-based numerical schemes for the computing of compressible flows resulting in popular family of schemes like AUSM [8,9], Roe [10] and HLL [11] schemes.

The ENO/WENO schemes are perhaps the most sophisticated among the latest flux-based schemes belonging to the class of TVD shock capturing methods [12–14]. The essential idea is to combine several lower order stencils using suitable local weights so as to obtain a locally higher order approximation to the numerical flux. However, these ENO/WENO schemes were found to be too complex and costly to implement in a practical multi-dimensional computing scenario as reported in [15]. A Weighted Compact Scheme (WCS) employing weighted (convex) combination of several compact implicit derivative approximations was reported in [16]. However, the scheme did not perform well for Euler test cases with shocks as a result of global dependency introduced via implicit nature of derivative approximations. Recently, a Modified Weighted Compact Scheme that essentially involves a weighted combination of WENO and WCS schemes is reported in [17]. The basic idea is to improve upon the shock resolution capabilities of WCS scheme and retain the high order in the smooth regions of the solution. In order to reduce the computational costs/overheads, global weights based on pressure and density only were employed for the WENO and WCS schemes. However, the choice of global weights reduces the accuracy in the smooth regions. In addition, far too many weight and mixing/blending functions with adjustable parameters are introduced that are difficult to optimize from problem to problem.

Among the more recent efforts towards the development of high accuracy schemes for both incompressible and compressible flows, the pioneering efforts of Sengupta et al. [18–20] are worth mentioning. For compressible flows governed by Euler/Navier–Stokes equations the basic approach of flux-vector splitting based on eigenvalues/eigenvectors of the inviscid flux jacobians was retained to represent the flow physics. Employing compact schemes with free parameters for spatial discretization, they have shown that the free parameters can be adjusted to yield optimal numerical properties like spectral accuracy, stability and dispersion relation preservation (DRP property) for model equations like the linear advection equation (LAE). Since the compact schemes introduce global dependencies they are likely to generate spurious oscillations/overshoots/undershoots, a feature known popularly as Gibbs' phenomenon, in the neighborhood of shocks in handling a

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