Computers & Fluids 119 (2015) 197–203

Contents lists available at ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

A consistent method for finite volume discretization of body forces on collocated grids applied to flow through an actuator disk

N. Troldborg *, N.N. Sørensen, P.-E. Réthoré, M.P. van der Laan

DTU Wind Energy, Department of Wind Energy, Technical University of Denmark, Risø Campus, DK-4000 Roskilde, Denmark

ARTICLE INFO

Article history:
Received 21 January 2015
Received in revised form 13 May 2015
Accepted 23 June 2015
Available online 2 July 2015

Keywords:
CFD
Actuator disk
Collocated grid
Finite volume
Discrete body force

ABSTRACT

This paper describes a consistent algorithm for eliminating the numerical wiggles appearing when solving the finite volume discretized Navier–Stokes equations with discrete body forces in a collocated grid arrangement. The proposed method is a modification of the Rhie–Chow algorithm where the force in a cell is spread on neighboring cells by applying equivalent pressure jumps at the cell faces. The method shows excellent results when applied for simulating the flow through an actuator disk, which is relevant for wind turbine wake simulations.

1. Introduction

The main advantage of using a collocated grid arrangement for finite volume based computational fluid dynamics (CFD) is that it is better suited for non-orthogonal grids. However, in contrast to a standard staggered arrangement, a collocated grid does not inherently ensure strong pressure–velocity coupling. The most common way to avoid the pressure–velocity decoupling is by using the interpolation scheme proposed by Rhie and Chow [1]. Over the years, a vast number of corrections have been proposed to the original Rhie–Chow interpolation scheme to remove its dependence on under-relaxation parameter and time steps [2–8]. However, these schemes are not capable of eliminating the non-physical wiggles in the flow field occurring as a consequence of discontinuous body forces or pressure jumps. In fact these interpolation schemes cannot properly handle any flows in which there are spatially varying body forces, but since the numerical wiggles are most pronounced near discontinuities the issue is most often addressed in connection with flows involving immersed like boundaries and actuator surfaces.

One way to avoid the wiggles is by smoothing the forces using a suitable smearing function. This approach was proposed by Sørensen and Shen [9] for transferring the forces from actuator lines (representing the blades of a rotor) to the computational mesh. In their work, the forces are distributed smoothly to the cells surrounding the rotor through a regularization kernel where the regularization function is a Gaussian. This method is simple to implement and is currently the most widely used in connection with actuator disk/line based methods for simulating wind turbine rotors [10–15]. The disadvantage of this method is that it depends on a free parameter, ϵ, which controls the amount of smearing. Previous studies have shown that the choice of ϵ does influence quantities such as the power performance of the wind turbine [16]. The parameter can be chosen to reflect the chord of the wind turbine blade but usually it is chosen as a compromise between diminishing the numerical wiggles and limiting the smoothing of the flow field [17]. Another drawback of the method is that it in principle involves evaluating an exponential function in the entire computational domain at each time step and thus may be rather computationally expensive. Furthermore, this approach is not fully consistent because it merely treats the symptoms of handling the body forces incorrectly and does not treat the origin of the wiggles, which is that the body force term is not discretized in the same way in the momentum and continuity equations.

Mencinger and Zun [18] presented a consistent discretization of the body forces, in which the body forces are effectively computed at cell faces and then interpolated to the cell center in a similar fashion as the pressure gradient is treated in Rhie–Chow interpolation. The proposed discretization rule is derived for a quiescent fluid subject to body forces but it is also shown to give good results for a moving fluid. However, the method is only applied for modeling the forces in two phase volume-of-fluids and cannot be directly applied for simulating the surface forces on an immersed

* Corresponding author.
E-mail address: niet@dtu.dk (N. Troldborg).

http://dx.doi.org/10.1016/j.compfluid.2015.06.028

like boundary without a method for transferring surface forces to volume forces. One way to achieve this is to replace the surface force with a volume force through a delta function [19], but this approach typically depends on a free parameter for representing the delta function discretely and thus suffers from the same disadvantage as the method by Sørensen and Shen [9].

Réthoré et al. [20,21] proposed a method equivalent to Mencinger and Zun, in which a discrete body force is first transformed into pressure jumps on the cell faces and then used to obtain the body forces in cell centers. They applied their so-called force allocation method for simulating an actuator disk in a uniform inflow and showed that it effectively removed the wiggles without the use of a regularization parameter. Furthermore, they validated and verified their results through a comparison with both analytical solutions as well as full rotor computations. Besides treating the body forces consistently, the main advantage of this method is that it offers a sharper representation of the actuator surface than methods using a regularization parameter.

However, as we shall see in the present work, the force allocation method of Réthoré and Sørensen [20] produces a discontinuous flow field near the actuator disk if its center axis is not aligned with the background grid. This problem turns out to be related to the way the pressure jumps are determined and is not a problem of the overall method itself. In the present work, we therefore present an improved method for determining the pressure jumps, which yields good results even when the rotor is not aligned with the grid.

2. Treatment of body forces

The starting point for the proposed method is the finite volume discretized equations of momentum and mass of an incompressible fluid with constant viscosity μ, which for a cell P read:

$$\rho\frac{\partial u_i^P}{\partial t}V^P + \sum_f C_f^P u_{f,i}^P = -\nabla p^P V^P + \mu\sum_f \nabla u_{f,i}^P \cdot \mathbf{n}_f^P S_f^P + f_i^P V^P \tag{1}$$

$$\sum_f C_f^P = 0 \tag{2}$$

where t, ρ and p are time, density and pressure, respectively. u_i^P and f_i^P denote the i-component ($i = x, y, z$) of the velocity and body force in cell P, respectively, while $u_{f,i}^P$ is the ith component of velocity on the cell's face f ($f = w, e, s, n, b, t$). V^P denotes the volume of cell P, whereas S_f^P and $\mathbf{n}_f^P$ are the area and outward pointing normal vector, respectively of the cell's face f. Finally, $C_f^P = \rho\mathbf{u}_f^P \cdot \mathbf{n}_f^P S_f^P$ is the flux of mass through cell face f of cell P.

The velocity and pressure are coupled via e.g. the Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) [22], which essentially is an iterative procedure where the velocity is first predicted by solving Eq. (1) and then corrected by solving Eq. (2). In order to solve these equations, it is necessary to find the velocity at the cell faces, which is not known a priori on collocated grids. One way to get these is to interpolate from cell centers to cell faces but this leads to the well known odd even pressure decoupling. To overcome this issue Rhie and Chow proposed to estimate the velocity at a cell face from Eq. (1) by computing the pressure gradient term directly at the cell faces and obtaining the remaining terms through interpolation from the cell centers. This approach only works well for flows without spatially varying body forces.

The method we propose here for handling body forces on collocated grids is based on the method by Réthoré and Sørensen [20]. Here a discrete body force is first transformed into pressure jumps on the cell faces, which are then used directly in the estimation of the face velocities in Eqs. (1) and (2) similarly to the way pressure is handled in Rhie/Chow interpolation. Thus, the idea is to express the ith component of the body force in cell P, f_i^P as the sum of pressure jumps on the cell faces, i.e.

$$f_i^P V^P = \sum_f n_{f,i}^P S_f^P \hat{p}_{f,i}^P \tag{3}$$

Here $\hat{p}_{f,i}^P$ is the pressure jump in the i-direction on face f of cell P and $n_{f,i}^P$ is the i-component of the corresponding outward-pointing unit normal vector, see Fig. 1. In case a neighboring cell also contains a body force, then the pressure jump on their common face is added together to form $\bar{\hat{p}}_{f,i}$, which is then used in the estimation of face velocities [20]. Taking the east face of cell P as an example, we get:

$$n_{e,i}^P \bar{\hat{p}}_{e,i}^P = n_{e,i}^P \hat{p}_{e,i}^P + n_{w,i}^E \hat{p}_{w,i}^E \tag{4}$$

where superscripts P and E refer to the cell in which the pressure jump is computed. Note that with the used syntax we have $n_{e,i}^P = -n_{w,i}^E$ and $n_{e,i}^P \bar{\hat{p}}_{e,i}^P = n_{w,i}^E \bar{\hat{p}}_{w,i}^E$.

The method used to convert body forces to pressure jumps, $\hat{p}_{f,i}$ can in principle be chosen arbitrarily as long as they fulfil Eq. (3) and that the final pressure jumps $\bar{\hat{p}}_{f,i}$ are used directly for the estimation of face velocities. In any case the pressure jumps are subsequently used for recomputing the body forces in the momentum equation at the cell center. Thus for cell P we get:

$$\bar{f}_i^P V^P = \frac{1}{2}\sum_f n_{f,i}^P S_f^P \bar{\hat{p}}_{f,i}^P \tag{5}$$

where the division by two implies that each neighboring cell of a face carry the pressure jump equally. Thus, the new body force $\bar{\mathbf{f}}$ is in practice equal to the original uncorrected body force $\mathbf{f}$ smeared over the nearest neighboring cells but in contrast to a standard smearing, the additional consistent use of the pressure jumps in the calculation of the fluxes, effectively removes the numerical wiggles.

2.1. Computing pressure jumps: original method

In the original work by Réthoré and Sørensen [20], a solution to Eq. (3) is obtained by assuming that the pressure jumps on each face of cell P scales according to its normal vector and surface area, i.e.:

$$\frac{\hat{p}_{f,i}^P}{S_f^P n_{f,i}^P} = \frac{\hat{p}_{f',i}^P}{S_{f'}^P n_{f',i}^P} \tag{6}$$

where index f and f' refers to different faces on cell P.

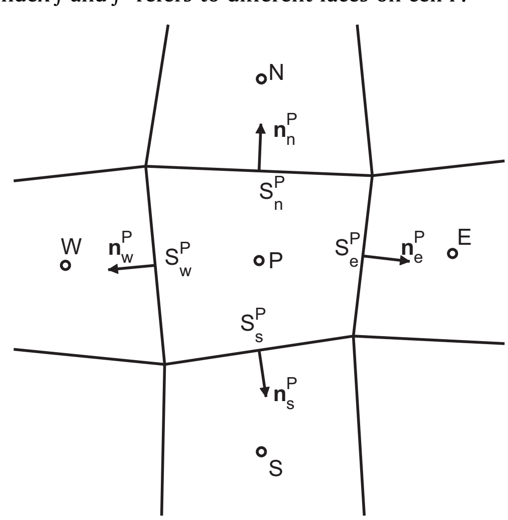

Fig. 1. Sketch of finite volume grid (in 2D) with definition of the used notation.