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Review

Multiple stable solutions in the 2D symmetrical two-sided square lid-driven cavity

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ABSTRACT

Stable solutions of a 2D symmetrical two-sided square lid-driven cavity are numerically determined with spectral accuracy. In addition to the expected symmetrical solutions, a set of two non-symmetrical solutions, mirror images of one another, are obtained for Reynolds number (Re) greater than a critical value, Re_1 by suitably eliminating one of the symmetrical solutions. The symmetrical solutions which are reported in this paper are obtained for $Re \leqslant 4000$ and are all steady. The non-symmetrical solutions are computed for large values of Re until these solutions become unsteady, at a second critical Re, Re_2, viz., for $Re \geqslant Re_2 > Re_1$. The transition from a non-symmetrical solution to its symmetrical counterpart upon reducing the Re below Re_1 is addressed. It is observed that the symmetric solutions are those which maximize the flow kinetic energy per unit input energy.

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1. Introduction

The driven-cavity problem has long been considered as the ideal model for benchmarking Navier–Stokes numerical solvers, in particular for assessing their capability to treat a configuration until its extremely non-linear, possibly turbulent, regime is predicted. This model is actually the simplest one for analyzing the flow which occurs in an important industrial process, viz., the chemical etching or the film coating. In spite of the numerous numerical computations which have been performed on this configuration, there are still several physical issues that are far from being fully understood. One can say that the quintessential problem on closed flows is the driven cavity; there is neither heat transport nor species transport, but just momentum transport with incompressibility.

Amongst the variety of configurations which can be defined for this driven-cavity problem, the ones where symmetries are imposed via the velocity boundary conditions are the most interesting. They indeed raise the question of the interaction between the flow symmetry and its stability, in addition to the intrinsic characteristic of any non-linear system, viz., the possibility of multiple solutions. This paper considers a 2D symmetric situation, the two-sided square driven cavity, wherein two walls facing one another move in their plane with the same imposed velocity, the other two walls being at rest.

Although this problem has received much attention, [13,4,2,1,7,3,20], there is no complete identification, so far, of all the stable solutions to this 2D simple situation, whether or not they are steady. Five steady solutions are now known, [4], when the Reynolds number Re does not exceed 2000. One of these solutions is self-symmetrical, and the four others are made of two pairs of non-symmetrical solutions, each solution in one pair being symmetrical to the other pair.

The present paper addresses these issues, i.e., the stability of the solutions, the transition from one steady state to another and the manner of the transition. The numerical method uses a time marching procedure. While such a method can only lead to stable solutions, steady or not, it allows us also to follow transitional behaviors when the flow experiences a jump from a solution to another. The symmetrical solutions have been computed for Reynolds number values going up to 4000. These solutions are all steady. A pair of non-symmetrical solutions, symmetrical to each other, has also been determined and computed beyond its transition to unsteadiness, i.e., for Reynolds number values as large as 14,000. It is observed that the non-symmetrical flow solutions correspond to a less efficient conversion of the input energy into kinetic energy of the flow.

2. Physical configuration and equations of the problem

Fig. 1 is a sketch of a specific two-sided lid-driven cavity, a square cavity of unit size. The cavity is filled with a fluid set into motion, on account of its viscosity, by the velocity $\vec{\mathbf{v}} = U(x)\,\hat{\mathbf{e}}_x$ which is imposed on the top and bottom rigid lids. The left and right walls are rigid and taken to be at rest. No-slip of the fluid is imposed along all the walls. This physical situation is therefore symmetrical about the centerline $z = 1/2$. The imposed lid velocity $U(x)$ is chosen so as $\vec{\mathbf{v}}$ and $\vec{\nabla}.\vec{\mathbf{v}}$ are not singular at the contact points of the driving lids with the fixed walls.

The fluid flow is described by the dimensionless momentum and mass balance equations, viz.,

$$Re\left[\frac{\partial\vec{\mathbf{v}}}{\partial t} + (\vec{\mathbf{v}}.\vec{\nabla})\vec{\mathbf{v}}\right] = -\vec{\nabla}p + \vec{\nabla}^2\vec{\mathbf{v}} \tag{2.1}$$

and

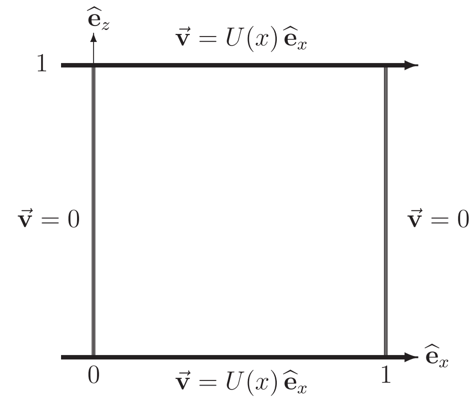

Fig. 1. The dimensionless 2D symmetric two-sided lid-driven cavity.

$$\vec{\nabla}.\vec{\mathbf{v}} = 0, \tag{2.2}$$

where the dimensionless variables are the velocity, $\vec{\mathbf{v}}(x,z) = v_x\,\hat{\mathbf{e}}_x + v_z\,\hat{\mathbf{e}}_z$, and the dynamical pressure, $p,\ t$ being the time coordinate. These equations are made dimensionless upon using characteristic scales, L for the length (physical size of the cavity), V for the velocity (driving-lid maximum velocity), $\Pi = \frac{\mu V}{L}$ for the pressure, μ being the dynamic viscosity of the fluid, and $\frac{L}{V}$ for the time scale. The Reynolds number is $Re = \frac{\rho VL}{\mu}$, where ρ is the fluid density.

These equations are closed by taking into account the no-slip conditions for the velocity which are indicated in Fig. 1. There is no boundary condition to impose on the pressure.

It is easy to check the following mirror-symmetry property about the z-centerline: if $\vec{\mathbf{v}}^{(1)}(x,\ z) = v_x^{(1)}(x,z)\,\hat{\mathbf{e}}_x + v_z^{(1)}(x,\ z)\,\hat{\mathbf{e}}_z$ and $p^{(1)}(x,\ z)$ are a solution of the problem (2.1) and (2.2) which satisfies the no-slip boundary conditions, then there exists another solution, symmetrical of the first one about the z-centerline, viz., $\vec{\mathbf{v}}^{(2)}(x,\ z) = v_x^{(1)}(x,\ 1-z)\,\hat{\mathbf{e}}_x - v_z^{(1)}(x,\ 1-z)\,\hat{\mathbf{e}}_z$ and $p^{(2)}(x,\ z) = p^{(1)}(x,\ 1-z)$. The Stokes flow, i.e., the solution of the steady linear problem posed with $Re = 0$, is unique and self-symmetric, implying that $v_x(x,\ z) = v_x(x,\ 1-z)$ and $v_z(x,\ z) = -v_z(x,\ 1-z)$. This leads to predict the existence of Navier–Stokes solutions which are also self-symmetrical. However non-symmetrical Navier–Stokes solutions are also allowed on account of the non-linearity of the problem. There is no systematical way to determine exhaustively all these solutions. One pair of non-symmetrical solutions, $\vec{\mathbf{v}}^{(1)}(x,\ z)$ and $\vec{\mathbf{v}}^{(2)}(x,\ z)$ will be identified and described in Section 5.

3. Numerical method

Eq. (2.1) is time integrated by using a usual second-order finite-difference scheme, the diffusion term being implicitly treated in time with the convective term explicitly evaluated. The resulting spatial problem is discretized by Chebyshev collocation based on N_x and N_z Gauss–Lobatto points along the x- and z-directions, respectively. The uncoupling between the velocity and pressure fields is performed from Eq. (2.2) by the method of Projection-Diffusion, [6]. It leads to an elliptic operator acting on the pressure which is solved by Successive Diagonalization, [16,12], a technique which is also used for inverting the Helmholtz problem posed for the velocity components. All the numerical details are presented in [14,11]. The numerical results are obtained via the nodal values (on a Gauss–Lobatto grid) of the polynomial approximation of the velocity and pressure fields evaluated at successive times. These polynomials are denoted by $\bullet^{(k)}(x,\ z) \equiv \bullet(t = k\,\delta t,\ x,\ z)$ where $\bullet(t,\ x,\ z)$ stands for the numerical approximation of $p,\ v_x$ and v_z. The numerical fields which are