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A low diffusion flux splitting method for inviscid compressible flows

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ABSTRACT

A low diffusion flux splitting method capable of capturing crisp shock profile and exact contact surface is presented. Here, the flux vector of the Euler equations is split into convective and pressure parts following Toro–Vázquez formulation. The low diffusive property of the present scheme is brought about by adding an anti-diffusion term to the pressure parts. This numerical method can be regarded as an improved version of Toro and Vázquez's flux splitting schemes (i.e. TV and TV-AWS) which are found to produce shock instabilities and carbuncle phenomena. Numerical results for several carefully chosen one- and two-dimensional test problems are investigated to demonstrate the accuracy and robustness of the proposed scheme.

1. Introduction

The high speed flow problems usually involve complex flow phenomena, such as strong shock waves, shock–shock interactions and shear layers [1]. Prediction of these problems requires robust, efficient and accurate numerical methods. Upwind methods are considered as the most appropriate numerical tools for predicting high speed flows, which are usually classified as flux difference splitting (FDS) and flux vector splitting (FVS) methods.

The FDS scheme is based on the difference between the decomposition of fluxes, constructed on either exact or approximate solutions of the local Riemann problem between two adjacent states [1]. These methods have been found to give good performance on capturing the discontinuities represented by linear as well as non-linear waves. One of the most efficient and robust approximate FDS methods is the HLL Riemann solver proposed by Harten et al. [2]. It approximates the solution of Riemann problem with two signal waves. Einfeldt [3], who proposed various ways of computing the wave speeds has shown that this scheme (denoted by HLLE) satisfies many important properties such as positivity and entropy conditions. However it cannot resolve contact discontinuities due to its highly dissipative behavior. Several attempts have been made to improve its capability to resolve discontinuities represented by linear waves. Einfeldt modified the HLLE scheme (i.e. HLLEM) to improve the resolution of contact discontinuity by reusing the information of contact discontinuity in terms of modifying the intermediate state [4]. However, it has been reported that the HLLEM is less accurate than the Roe's flux scheme which captures the isolated stationary contact perfectly. Park [5] analyzed the dissipation mechanism of the HLLEM scheme and discussed the cause of inaccuracy at the contact discontinuity. He proposed another improved version of the HLLE scheme which was capable of capturing contact discontinuity accurately. Unfortunately, both the modified versions of the HLLE scheme are found to produce unacceptable results like low frequency post shock fluctuations in case of slowly moving shock, carbuncle phenomenon, odd–even decoupling and kinked Mach stem on certain occasions. The HLLC method is another contact-preserving HLL-type scheme which also possesses all desirable properties like positivity and entropy conditions [6]. However, it also encounters several shock instability problems mentioned previously.

The FVS methods have been found to be free from shock instabilities and carbuncle phenomena. These methods rely on a decomposition of the flux vector into upstream and downstream components according to the sign of the propagation of the associated waves. They are found to perform perfectly on capturing steady discontinuities represented by nonlinear waves, which include shocks. However, they are not effective in resolving intermediate characteristic fields and this insufficient badly affects the correct resolution of contact waves, material interfaces, shear waves, vortices and ignition fronts [7]. A lot of attempts have been made to improve the capability of the FVS methods to capture contact discontinuities while being free from numerical shock instability problems. Another class of flux vector splitting type methods has been attracting extensive attentions during the past two decades, which combines features from the flux vector splitting and Godunov approaches. One typical example is Liou and

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Steffen's AUSM scheme [8]. Further developments can be seen in Refs. [9–11]. Another flux vector splitting scheme analogous to the AUSM scheme is that presented by Zha and Bilgen [12]. The total flux vector for these methods is split into a convective component and a pressure component following the guideline that the velocity and pressure should be separated to consider their characteristics representing the physics of the convection and waves [13]. Recently, Toro and Vázquez-Cendón proposed two flux splitting methods which are based on a new type of flux splitting formulation [7]. The numerical schemes, named TV and TV-AWS, are reported to enjoy a desirable property: recognition of contact and shear waves in general and exact preservation of isolated stationary contacts. However, their methods are also found to produce shock instabilities and carbuncle phenomenon in the case where strong shock waves exist. The difference among Liou and Steffen's, Zha and Bilgen's and Toro and Vázquez's splitting methods is in the convective quantity in the energy equation: total enthalpy in Liou–Steffen, total energy in Zha–Bilgen and kinetic energy in the Toro–Vázquez splitting. Considering the different propagation mechanism of the convective terms and pressure terms, Toro and Vázquez's splitting seems to be more reasonable compared with other two methods because all the pressure terms are included in the pressure flux.

This paper is concerned with a new flux splitting scheme based on Toro–Vázquez splitting, which is supposed to give more accurate, robust and even efficient solutions for inviscid compressible flows when compared with Toro and Vázquez's original methods (i.e. TV and TV-AWS) [7]. What should be noted is that the new flux splitting scheme (denoted by present) in this paper means a new numerical scheme to treat Toro–Vázquez splitting, while the TV and TV-AWS represent TV and TV-AWS schemes for the Toro–Vázquez flux splitting. In the new flux splitting scheme, the convective parts of the flux vector are dealt with a modified version of Mandal and Panwar's approach [14]. The pressure components of the flux vector are evaluated based on the HLL-type Riemann solver. In order to make the numerical scheme low diffusive, an anti-diffusion term is added to the pressure parts in a way similar to the HLLEM method. The anti-diffusion coefficients are carefully designed to make all the dissipation vanish at the contact discontinuity. So the numerical method proposed is also more accurate than other HLL-type Riemann solvers such as the HLL and its modified version HLLEM scheme. Results of massive numerical tests show that the new method is also free from shock instabilities and carbuncle phenomena which confuse most of the shock-capturing schemes that are designed to preserve contact discontinuities. In this paper, we fix our attention on the first-order case, as there are several approaches to extend first-order methods to high order of accuracy.

The paper is structured as follows. Section 2 briefly reviews the Euler equations and Toro–Vázquez splitting method. Section 3 presents a detailed description of the new low diffusion scheme. Detailed analysis on the numerical dissipation mechanism of the new scheme is given in Section 4. In Section 5 we present numerical results for several carefully selected test problems to assess both robustness and accuracy of the scheme proposed in this paper. Finally, conclusion remarks are given in Section 6.

2. Preliminaries

2.1. Governing equations

The two-dimensional Euler equations written in integral formulation are as follows:

$$\frac{\partial}{\partial t}\int_{\Omega}\mathbf{Q}\,d\Omega+\oint_{\partial\Omega}\mathbf{F}\,dS=\mathbf{0} \tag{1}$$

where $\mathbf{Q}$ and $\mathbf{F}(\mathbf{Q},\mathbf{n})$ are the vectors of conservative variables and conservative fluxes, both given by

$$\mathbf{Q}=\begin{bmatrix}\rho\\ \rho u\\ \rho v\\ E\end{bmatrix},\quad \mathbf{F}(\mathbf{Q},\mathbf{n})=\begin{bmatrix}\rho U\\ \rho uU+n_xp\\ \rho vU+n_yp\\ U(E+p)\end{bmatrix}. \tag{2}$$

The contravariant velocity U is defined as the scalar product of the velocity vector $\mathbf{u}$ and the unit normal vector $\mathbf{n}$, i.e.,

$$U\equiv\mathbf{u}\cdot\mathbf{n}=n_xu+n_yv \tag{3}$$

The equation of state has the form as follows:

$$p=(\gamma-1)\rho\left[E-\frac{1}{2}(u^2+v^2)\right] \tag{4}$$

where the specific heat ratio γ is 1.4 for a perfect gas.

The finite volume discretization of (1) can be written as

$$\mathbf{Q}_{i,j}^{n+1}=\mathbf{Q}_{i,j}^{n}-\frac{\Delta t}{\Omega_{i,j}}\sum_{m=1}^{N_F}\mathbf{F}_m\Delta S_m \tag{5}$$

ΔS_m is the edge length, N_F is the number of edges enclosing the 2D finite volume $\Omega_{i,j}$.

2.2. The Toro–Vázquez splitting

In the present approach, the flux vector $\mathbf{F}(\mathbf{Q},\mathbf{n})$ is split into a convective part and a pressure part following Toro and Vázquez's formulation

$$\mathbf{F}(\mathbf{Q},\mathbf{n})=\mathbf{C}(\mathbf{Q},\mathbf{n})+\mathbf{P}(\mathbf{Q},\mathbf{n}) \tag{6}$$

with

$$\mathbf{C}(\mathbf{Q},\mathbf{n})=U\begin{bmatrix}\rho\\ \rho u\\ \rho v\\ \rho(u^2+v^2)/2\end{bmatrix},\quad \mathbf{P}(\mathbf{Q},\mathbf{n})=\begin{bmatrix}0\\ n_xp\\ n_yp\\ U(\rho e+p)\end{bmatrix}. \tag{7}$$

It should be noted that no pressure terms exist in the proposed convective flux $\mathbf{C}(\mathbf{Q},\mathbf{n})$. All pressure terms from the total flux vector $\mathbf{F}(\mathbf{Q},\mathbf{n})$, including that of the total energy E, are included in the pressure flux $\mathbf{P}(\mathbf{Q},\mathbf{n})$ [7]. The information of the convective terms is transported in the direction as the velocity U goes, and the information of the pressure terms propagates in all directions at the speed of sound a which is introduced as

$$a^2=\frac{\gamma p}{\rho} \tag{8}$$

3. The low diffusion scheme

We give a detailed description of the new low diffusion scheme here. Since the total flux vector has been split into a convective component and a pressure component, the numerical treatment for each part has been described separately. We start with the convective system.

3.1. Evaluation of the convective flux

The flux vector $\mathbf{C}(\mathbf{Q},\mathbf{n})$ is computed by a modified version of Mandal and Panwar's method [14]. Here, we give a brief description of their technique used to deal with the convective flux. For a detailed description of the numerical treatment of the convective flux see the original paper of Mandal and Panwar [14].