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Optimised active flow control for micromixers and other fluid applications: Sensitivity- vs. adjoint-based strategies

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ABSTRACT

Active flow control to improve performances and efficiencies of technical fluid systems is of significant importance in many engineering applications. In this work, we investigate two optimisation strategies that are commonly used for systems which can be described by partial differential equations, namely the adjoint-based and the sensitivity-based approach. The approaches are reviewed and their merits and disadvantages are discussed. We employ both strategies to minimise the separation of a laminar flow over a backward-facing step and critically compare the outcome. It turns out that both strategies are able to considerably reduce the reattachment length compared to the uncontrolled case. Nevertheless, the sensitivity-based approach allows for a simpler implementation and a less complex solution process and therefore appears attractive for engineering applications. The sensitivity-based approach is implemented for the active flow control in an electrokinetic micromixer and various numerical experiments are performed. It is shown that the sensitivity-based optimisation leads to very high mixing degrees in short operation times.

1. Introduction

Nowadays, the ultimate goal for many research and development projects concerned with computational fluid dynamics (CFD) is related to the optimisation and the optimal control of the system/process. In this context, the solution process is usually much more complex than a purely forward-in-time simulation which only results in a descriptive or predictive view of the system for a given set of process parameters. Optimisation and optimal control require an iterative process which adjusts the control variables such that the process approaches an improved mode of operation. This improvement has to be diagnosed and verified to evaluate the success of the optimisation strategy. Especially in fluid mechanics, optimisation strategies are becoming important for the field of active flow control. Active flow control refers to the use of transient and often periodic perturbations that are introduced into the flow field by so-called actuators. The nature of the actuator is manifold and comprises moving surfaces/boundaries, fluidics (injection or suction), plasma or other forces such as electro- or magneto-dynamic. Examples for which active flow control proved to be most promising are control of separation, mixing and combustion among others. Comprehensive review on the topic is available in Refs. [1,2].

The adjoint- and the sensitivity-based approach are two optimisation strategies which are commonly used for systems which can be described by a set of partial differential equations (PDE). Both approaches have been engaged for shape optimisation and for active flow control. For example, stationary and rotating turbomachinery blades were optimised based on a continuous adjoint method with the objective function derivatives based on boundary integrals [3]. A good example for optimal active flow control was given by Zhang et al. who employed a nonlinear adjoint-based approach to suppress the flow separation in the wake of a cylinder by an electromagnetic force [4]. A sensitivity-based approach was used by Leclerc and co-workers for the control of the laminar vortex shedding past a circular cylinder by suction and blowing [5]. Another active flow control example demonstrating the suitability of sensitivity-based functions is the optimisation of a flapping airfoil as given by Soued et al. [6]. However, there is practically no work available in literature which compares both the adjoint- and sensitivity-based approach with respect to applications in active flow control.

The present work is focused on optimal control within the framework of numerical simulation of PDEs. We analyse the two

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optimisation workflows by means of a laminar flow over a backward-facing step. For this study, the Navier–Stokes equations (NSE) are solved with the control objective of suppression of flow/boundary layer separation. Based on the outcome of this optimisation, we choose a strategy for another objective of this work. That is, we investigate the optimisation of an active (electrically-excited) flow in an electrokinetic mixer which can be used in microfluidic devices such as Lab-on-a-Chip (LOC).

In general, the optimisation problems in this work can be summarised as: Let y denote a set of state variables and u be a control variable of any kind. The system of PDEs describing the fluid flow is given by the abstract equation $\partial_t y + A(y, u) = 0$. Furthermore, a cost or objective functional $J(y, u)$ is defined that has to be minimised such that the system of partial differential equations is fulfilled. The optimisation methods discussed in this work can generally be distinguished by the determination of the gradient of the objective functional.

Assuming a time dependent setting, methods that rely on the solution of the adjoint equations require a forward-in-time solution of the state equations and a backward in-time solution of the adjoint equations. In contrast, sensitivity-based methods only march forward in time. The reverse time character of the adjoint equations is partially caused by the integration with respect to time and space in order to remove derivatives from the arbitrary variations of the adjoint state (cf. Section 3.1 or Ref. [36]). In case of nonlinear problems, another challenging task is the fact that the state variables have to be readily accessible to the adjoint equation solver. However, since the adjoint equation solver marches backward in time, the state variables for every time step must be stored. For large scale problems, this requirement generates very large data sets which can be intractable in practice or which become a real bottleneck for the overall solution process.

This paper is organised as follows: In Section 2, we discuss the motivation of the present work and introduce both active flow control problems that we investigate. That is, the flow over a backward-facing step and an electrokinetic mixer concept which is based on the electrical excitation of secondary flows in a meandering microchannel. For both problems, the respective mathematical models/governing equations which allow for the simulation of the physical phenomena are presented. Additionally, we introduce the simulation domains, boundary conditions, and the simulation parameters which are used for the active flow control over a backward-facing step and the micromixer operation. We proceed with Section 3 which represents the general framework for adjoint- and sensitivity-based optimisation under the constraints of time-dependent PDEs. Here, we discuss the main differences in computation of the gradient of the objective functional and their consequences for implementation and computational costs. This article is continued with the numerical experiments as described in Section 4. That is, we apply both optimisation approaches to the backward-facing step example and critically compare the outcomes. Based on the findings, we choose the sensitivity-based optimisation approach for the flow control of the electrokinetic micromixer. Finally, we conclude this article by a brief summary and by identifying directions for further research.

2. Model and problem formulation

In this section, the foundations for both optimisation problems are laid out. Some essentials of the flow over the backward-facing step and of the electrokinetic micromixer are discussed and the governing equations and respective boundary conditions are identified.

2.1. Backward-facing step

The flow over a backward-facing step is one of the classic problems in fluid mechanics and it has been an object of numerous investigations. This is due to the considerable technical relevance, e.g. for various fluidic elements or cooling of turbine blades, and even though the geometry is relatively simple, the flow structure is rather complex. Generally, the streamlines which approach the step are more or less parallel to the wall while the flow separates at the upper edge of the step (convex corner). Moffatt's fundamental studies of flows in sharp corners showed that even for creeping flows, vortices should be formed as well in the lower edge of the step (concave corner) [7]. The length which is required for the flow to reattach to the wall depends on the Reynolds number. An exact analytical solution of the backward-facing step flow problem is not yet available, and the application of potential theory neither recovers the convex corner separation nor the concave corner recirculation [8]. Hence, only experimental and numerical strategies can contribute to a detailed insight into the flow. A significant amount of work has been devoted to the flow over the backward-facing step and various details of the flow topologies such as detachment and reattachment length have been determined depending on aspect and expansion ratio of the step and the Reynolds number of the flow. A comprehensive review on relevant literature is available in reference [9].

Since the separation and recirculation of the flow cause pressure losses, vibrations/noise and influence heat transfer, there are multiple efforts to suppress them by active flow control. Here, different means have been employed including an oscillating flapping foil in the recirculation area of the flow whose effectiveness increased with frequency and amplitude of oscillation. A maximal reduction of the reattachment length as much as 70% of the uncontrolled flow value was observed [10]. Chun and Sung used a sinusoidally oscillating jet starting from a thin slit at the convex corner. They observed a minimum of the reattachment length for a higher localised actuation. For lower localised actuation, however, the recirculation area was increased [11]. A comparable setup was used by Wengle et al. who used loudspeakers at the convex corner to manipulate the flow by a low-amplitude harmonic blowing/suction excitation. A reduction of 1/3 of the reattachment length was observed for the highest excitation frequency (50 Hz) investigated [12]. Sano et al. used continuous suction through a slit at the bottom corner of the step as means of active flow control. The direction of the suction was either horizontal or perpendicular to the main flow. It was found that the reattachment length gets smaller when the suction to flow ratio increases. Also, no significant difference between horizontal and vertical suction was observed [13]. A comparable work has been conducted by Uruba et.al. who found that both horizontal continuous suction, as well as blowing, can be used to reduce the reattachment length of the separation zone [14].

We consider the active control of the three dimensional (3D) flow over a backward-facing step as a model problem in order to test the adjoint- and sensitivity-based approach; similar considerations for the two-dimensional case can be found in [15]. In detail, we perform optimisations in order to suppress the separation and minimise the reattachment length of the flow. The active flow control is realised by the manipulation of the boundary conditions located at the back-side of the step close to the convex corner (cf. area Γ_c in Fig. 1). In practice, our approach can be realised by an array of orifices which allows for injection or suction of streams/jets of fluids in a desired direction.

2.1.1. Mathematical formulation of the backward-facing step problem

In this section, the computational framework for the flow over the backward-facing step is introduced. The computational domain