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A generic framework for design of interface capturing schemes for multi-fluid flows

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ABSTRACT

A generic methodology for the design of interface capturing schemes for immiscible multi-fluid flows using the VOF (Volume-Of-Fluid) approach is outlined. Interface capturing schemes are devised as a blend of high-resolution and compressive schemes and their efficiency and accuracy are dependent on the choice of the constituent schemes and the blending function. On the basis of a set of design principles proposed in this work, we show that interface capturing schemes proposed in literature may be encompassed into a single class of GPL (Generalised Piecewise Linear)-κ schemes allowing for a unified approach for development of such schemes. This methodology is used to de-construct four existing schemes and to propose two new schemes for interface capture in an unstructured finite volume framework. The schemes are tested on challenging advection problems using both structured and unstructured grids to evaluate their performance. The new schemes perform as well as existing schemes and even outperform them on unstructured grids, while also exhibiting near Courant number independence for all test problems and grid topologies. The CUIBS (Cubic Upwind Interpolation based Blending Scheme) proposed in the study shows the best performance among all interface capturing schemes discussed herein and is applied to the study of immiscible binary fluid flows. Numerical simulations of viscous sloshing in a tank and Rayleigh–Taylor instability demonstrate the ability of the CUIBS scheme to resolve fluid–fluid interfaces with minimal diffusion.

1. Introduction

The success of numerical simulations of incompressible immiscible fluid flows depends on the ability of the algorithm to resolve the fluid interfaces accurately and preserve their sharpness. The Volume-Of-Fluid (VOF) method [1] is among the approaches employed for such simulations owing to its robustness and conservation properties. This approach involves solving advection equation(s) for the volume fraction(s) that define the interface(s) and utmost care needs to be taken to design stable and robust convective schemes that result in minimal numerical diffusion. While effective geometric methods to locally reconstruct the interface have been considered in literature [2,3], their implementation on unstructured meshes is a major challenge. The need for advection schemes that preserve a sharp interface without compromising on solution accuracy and allows for easy implementation on arbitrary mesh topologies leads to the concepts of interface capturing schemes. Interface capturing schemes are constructed by combining high-resolution schemes with compressive schemes, using a suitable blending function, thereby obviating the need for any geometric reconstruction of the interfaces. Among the earliest works in this regard are the HRIC scheme of Muzaferija et al. [4] and the CICSAM scheme proposed by Ubbink and Issa [5]. A prominent drawback of both these methods is the deterioration of their performance with Courant number necessitating the use of smaller values of Courant number. Darwish and Moukalled [6] developed the STACS scheme to successfully overcome this drawback and subsequently proposed a family of transient interface capturing schemes referred to as TICS family of schemes [7] that combine a bounded high-order transient scheme with a bounded high-order compressive scheme. Gopala and co-workers [8] proposed a scheme akin to the Inter-Gamma scheme of Jasak and Weller [9] and investigated its utility for free-surface flows. Walters and Wolgemuth [10] proposed the Bounded Gradient Maximization scheme which involves a cell-based flux limiting with a suitable weighting factor while Xiao and co-workers [11] developed the THINC scheme based on the tangent function to compute volume fraction flux that ensures oscillation-free solution for fluid interface problems. Notable recent attempts in this direction are the

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FBICS scheme [12] and the HiRAC scheme [13], with the latter using a modified advection equation with an artificial compression term to ensure sharp resolution with stability. Most of these schemes have been applied to time-dependent interfacial flows and have proven successful in resolving the complex flow dynamics. It must be remarked that there exist other approaches for multi-fluid and multiphase simulations in literature, most notably the Level Set (LS) approach by Osher and Sethian [14] where the interface is defined by a zero distance function and immiscible fluids on either side of interface are distinguished by positive and negative values of the distance function. While many researchers have successfully employed the LS approach in solving a wide variety of problems [15], its applications have mostly been restricted to structured Cartesian meshes and extension to unstructured meshes is not straightforward. An interesting approach to solving immiscible flows that has received significant attention is the diffuse interface technique [18], where the Navier–Stokes equations are solved in conjunction with convective Cahn–Hilliard equation. The method is conservative in nature but the interface needs special treatment to maintain its sharpness. Lattice Boltzmann Methods (LBM) have also been used for efficient simulation of multi-fluid flows which include studies of Rayleigh–Taylor instability in titled channel [16] and viscosity stratified flows [17].

The focus of the present work is on the VOF method with interface capturing as a methodology to handle multi-fluid immiscible flows on unstructured meshes in two dimensions. We emphasise that while several interface capturing schemes have been proposed in the past and are in use, there is no definitive framework to analyse these schemes for possible improvements and development of newer schemes. While almost all interface capture schemes are constructed as a blend of high-resolution and compressive schemes, there are no clear guidelines for the specific choices in open literature. Inspired by the unified approach for convective schemes introduced in [19] which brought several different higher-order convection schemes under a common framework, the authors have attempted to devise a comprehensive set of design principles for evaluation and development of interface capturing schemes for multi-fluid flows in this work. The rest of the paper is organised as follows. Section 2 briefly discusses the VOF approach and solution of the interface advection equation. High-resolution schemes and boundedness are discussed in Section 3 and the design principles for interface capturing schemes are proposed in Section 4. Examination of existing interface capture schemes and development of novel alternatives constitute Sections 5 and 6 and their performance for typical advection problems is the focus of Section 7. Section 8 describes the use of new interface capturing schemes for practical engineering problems such as liquid sloshing and Rayleigh–Taylor instability. A summary of the contributions of the study and possible future directions are given in Section 9.

2. Interface advection equation

The Volume of Fluid (VOF) approach captures the interface by solving a transport equation for the scalar volume fraction. The interface between any two fluids which are immiscible will convect as a passive scalar and therefore satisfies a hyperbolic equation that reads,

$$\frac{\partial \phi^k}{\partial t} + \mathbf{u} \cdot \nabla \phi^k = 0 \tag{1}$$

The quantity ϕ^k represents the fraction of the kth fluid contained in the cell, where the fluids are arranged in increasing order of densities. Therefore, for the case of k immiscible fluids, this approach solves $(k-1)$ advection equations for as many volume fractions where the velocity field is determined from the solution to momentum equations. In this study, we restrict ourselves to binary fluid flows ($k = 2$) which are immiscible as well as incompressible. Consequently, we now solve for a single volume fraction and the discrete counterpart of the advection equation in an unstructured finite volume framework may be written in conservative form as,

$$\Omega_c \frac{3\phi_c^{n+1} - 4\phi_c^n + \phi_c^{n-1}}{2\Delta t} + \sum_{f \in C} \phi_f^{n+1} U_f^n \Delta s_f = 0 \tag{2}$$

where Ω_c is the volume (or area in two-dimensions) of the cell C, ϕ_c is the cell-averaged value of the volume fraction in that cell, ϕ_f denotes the value at the midpoint of faces constituting the cell, U_f is the normal velocity at the faces constituting the cell and Δs_f is face area. The value of the volume fraction obtained in each cell is limited to lie in the range [0,1]. It must be remarked that $\sum_{k=1}^{2} \phi_c^k = 1$ and that the divergence-free condition is employed to recast the advection equation in its conservative form. The choice of second-order accurate backward differencing scheme (where the superscript now denotes the time level) is chosen to achieve time-accurate solutions for unsteady flows that are routinely encountered with immiscible binary fluids. The advection equation is solved assuming a constant value for the velocity field at a particular timestep, and is known from the solution to the momentum equations in the previous time-step. This approach of freezing the velocity field linearises the interface advection equation and the resulting system of linear algebraic equations (at a discrete level) are solved implicitly using an ILU-preconditioned GMRES approach [20]. It must be however remarked that for the tests carried out in Section 7, the (analytical) velocity field is prescribed directly, rather than computed as the solution to the momentum equations.

3. High resolution schemes and convection boundedness criterion

The solution to the interface advection equation necessitates a robust convection scheme that ensures sharp resolution of the discontinuity. First order schemes are robust but they are overly diffusive and smear the interfaces. While higher-order (second-order or more) accurate schemes appear to be a natural solution, they suffer from loss in monotonicity as a consequence of the Godunov Barrier Theorem [21]. The development of high-resolution schemes, which are at least second-order accurate with solution boundedness, has been a subject of research over the years. These schemes overcome the barrier theorem by introducing non-linear "flux limiters" to achieve solution monotonicity. Thus, the face value ϕ_f of the volume fraction on the uni-dimensional grid shown in Fig. 1(a) may be determined as,

$$\phi_f = \phi_C + \frac{\phi_C - \phi_U}{2} \cdot \gamma(r)$$

where ϕ_C and ϕ_U are values of volume fraction at cell centroid of the upwind and far upwind cell respectively. Here, $\gamma(r)$ represents the flux limiter (FL) which depends on the gradient ratio r, which

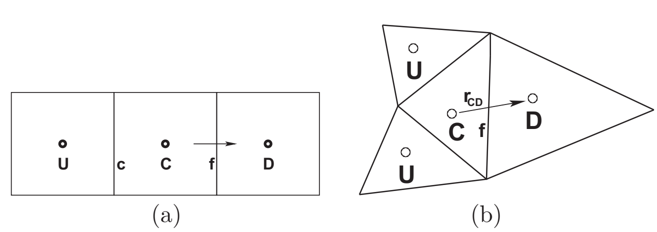

Fig. 1. Grid topology (a) structured mesh and (b) unstructured mesh.