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A nonoverlapping heterogeneous domain decomposition method for three-dimensional gravity wave impact problems

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ABSTRACT

A 3D heterogeneous flow model which combines incompressible viscous flow and potential flow both with free-surface boundaries is developed and solved numerically, using finite element method and boundary element method, respectively. The coupled model is solved based on a nonoverlapping domain decomposition method, which reduces the problem to the coupling interface. At the interface, the problem is equivalent to Bernoulli's equation consisting of mappings from potential/velocity to velocity/pressure. These mappings enable us to design the scheme following the Dirichlet–Neumann method for nonoverlapping domain decomposition, and lead to a staggered scheme. We also discuss data transfer as well as free surface reconstruction at the interface. Errors introduced by the surface reconstruction technique are examined and discussed in detail. The errors from the first-order staggered scheme is also studied. A comparison of numerical prediction with experimental data that the proposed method performs efficiently and accurately for wave impact problems.

1. Introduction

Gravity wave impact problems have been the subject of investigation in the past several decades, due to their challenging highly nonlinear nature and importance in offshore engineering [1], naval architecture [2] and coastal engineering [3]. Because of the transient nature of the problem and the arbitrary, complex geometry of the structures in industrial applications, a time domain study in physical space is required. Accompanied by advances in numerical methods, techniques in modeling free-surface flows in the time domain, such as the one pioneered by Longuet-Higgins and Cokelet [4] in tracking free surface using the mixed-Eulerian–Lagrangian method (MEL), have been gaining a steady interest. An integrated simulation environment for wave-structure interaction, often referred to as a numerical wave tank (NWT), assembles techniques of modeling wave generation, propagation, dissipation and interaction with structures. The past three decades have witnessed extensive development and studies on this topic. The modeling technique introduced in this paper is part of an on-going effort in the development of a robust and practical NWT.

Regarding the modeling of flows in a NWT, two approaches exist for the time domain wave simulation. The first and also the most popular one, is to use the homogeneous flow, by which a single model is used to describe the entire flow domain. The model could be viscous flow, governed by the Navier–Stokes equations (NSE), such as the work by Lin and Liu [5], Park et al. [6], Yuan and Wu [7], and Li et al. [8]. It could also be a simplified model such as potential flow (PF) or Boussinesq model, such as the work by Beji and Battjes [9], Grilli et al. [10], Ryu et al. [11], Liu et al. [12] and Nimmala et al. [13]. An extensive review of using PF solutions to NWT using the boundary integral equation (BIE) approach is given by Kim et al. [14].

In the second approach, the flow domain is decomposed into multiple *subdomains*, and in each subdomain a different flow model is adopted. This is sometimes referred to as a heterogeneous flow model [15, chap. 8]. The motivation of this approach is of two fold. First, the development of a large scale solver like NWT is often based on legacy codes, frequently originated from other purposes. For example, the wave generation/dissipation and viscous flow solution techniques have separate origins in coastal engineering and aerospace engineering, respectively. It is pointed out recently that the approach of combining code bases from different fields is the most common path taken by multiphysics solver developers, because it reduces development overhead and takes advantage of previous validations [16]. In the context of a NWT, such a multiphysics model often contains a NSE solver coupled with simplified models, such as a PF or a Boussinesq model, with the

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NSE describing the "near field" such as flow near a body or in the shoaling zone, and the simplified models describing the "far field". The second motivation of a heterogeneous flow model is to reduce computing cost. It is well known that though a NSE numerical solver captures viscous and vortex effects that cannot be computed in simplified models without empirical modeling parameters (see e.g., [17]), its computing cost is rather high, due to the nonlinearity and the propensity for instability induced by the incompressibility condition. On the other hand, for far field wave propagation, simplified models are sufficient and more efficient, as validated by numerous works in the past century (see, e.g., [18,19]).

Because of the spatial decomposition the flow domain, the modeling problem can be cast into the framework of (heterogeneous) domain decomposition (DD) methods. Early works on numerical applications of the DD methods include Bjørstad and Widlund [20], Cambier et al. [21], Dinh et al. [22], Dryja [23] and Glowinski et al. [24], among others. Recent overviews on this topic include the reviews by Xu and Zou [25], Xu [26], the books by Mathew [27], Quarteroni and Valli [15], Toselli and Widlund [28] and Wohlmuth [29]. In particular, the well-posedness of the problems of NSE coupled with Stokes flow or Oseen flow have been studied by Fatone et al. [30,31], Feistauer and Schwab [32,33], and Schenk and Hebeker [34].

Examples of the free surface flow numerical solvers based on the DD approach include the work by Campana et al. [35], Iafrati and Campana [36], Colicchio et al. [37], Sitanggang and Lynett [38], Kim et al. [39], Hamilton and Yeung [40], and Zhang et al. [41]. Among these works, all except [40] are 2D models. In [40], a non-overlapping coupled 3D solver based on the scheme proposed by Iafrati and Campana [36] is reported. In their study, the solver adopts the method of Green's function for the far field and a pseudo-spectral method for the viscous subdomain. Since the Green's function used is specifically developed for an axial symmetric domain as in the problem of the interaction between waves and a cylindrical structure, it is very efficient but also limited in application. Moreover, the solver is limited to a linearized free-surface boundary, and no wave generation is incorporated in the far field model.

In this paper we present the progress made toward a 3D DD solver, based on our 2D development and theoretical study, reported in Zhang et al. [41,42], respectively. In particular, the relationship between the coupling scheme and the Dirichlet–Neumann (D–N) method for a homogeneous elliptic problem [20,43] is discussed in Zhang et al. [41]. The scheme is related to the solution for the Schur's complement formulation of the coupled problem, as discussed in Zhang et al. [42]. In this paper, we follow this D–N-like approach, to study a 3D potential flow-viscous flow heterogeneous model, with nonlinear free surface tracking/capturing capability. The PF subdomain is solved using a boundary element method (BEM), whereas NSE subdomain is solved using a finite element method (FEM). The BEM solver is based on the work of Grilli et al. [10,44], Fochesato and Dias [45], and Nimmala et al. [13]. The BIE from the Laplace equation is discretized using a collocation BEM, and the linear system is solved using the generalized minimal residual method (GMRES) accelerated by the fast multipole method (FMM). On the other hand, the FEM solver for the NSE subdomain is based on a monolithic formulation and a pressure segregation scheme, proposed by Soto et al. [46–48] and Codina [49–51]. In this scheme, P1–P1 tetrahedron elements are used to discretize the domain. This type of element does not meet the Ladyzhenskaya–Babuška–Brezzi (LBB) condition [52–54], and the scheme provides stabilization toward pressure.

In the current study, we first further develop a 3D algorithm and implement it to the 3D NWT model. This includes introducing techniques for the interpolation and free surface matching at the interface (Section 3.2 and 3.3). Moreover, a numerical analysis for the error due to free-surface matching is performed on solitary wave propagation (Section 3.3). To further improve the mathematical robustness of our method, in the current study we further interpret the DD method using the Helmholtz decomposition, and explore the nature of the matching conditions as well as the corresponding operators (Section 4). A comparison of computing time is also shown to demonstrate the efficiency of the proposed method which was not examined in the previous 2D studies.

The rest of this paper is organized as follows. In Section 2 we describe the coupled flow model. In Section 3, we introduce the DD scheme based on two mappings, corresponding to the PF and NSE solution, respectively. We also discuss numerical techniques for data transfer and free surface reconstruction near the interface. In Section 4, we explore the mathematical foundation of the coupling scheme using the Helmholtz decomposition. In Section 5, two numerical examples are reported to validate the 3D DD model. In particular, we discuss spatial and temporal convergence of the method, as well as its efficiency in terms of computing time. Further discussions and conclusions are presented in Section 6.

2. Heterogeneous flow model

In our DD model, as shown in Fig. 1, the flow domain $\Omega \subset \mathbb{R}^3$ is decomposed into two non-overlapping subdomains Ω_{NSE} and Ω_{PF}, within which incompressible viscous flow and potential flow models are adopted, respectively. $\Gamma = \partial\Omega_{\mathrm{NSE}} \cap \partial\Omega_{\mathrm{PF}}$ is the interface between the subdomains. Though only the case of two-subdomain decomposition is presented in this paper, such philosophy applies to cases with a more complex decomposition. In wave-structure interaction applications, Ω_{PF} denotes the "far field" and Ω_{NSE} the "near field" of the domain, and the latter interacts with the structure domain(s). In this paper we focus on the initial impact of the waves, and the numerical tests show that it suffices to model the structures as rigid. Though Γ could be a general surface (2D manifold), in our implementation a vertical surface in the Eulerian representation is used for simplicity.

The governing equations for our heterogeneous model are

$$\rho\frac{D\boldsymbol{u}}{Dt} = -\nabla p + \mu\nabla^2\boldsymbol{u} + \boldsymbol{f}, \quad \forall(\boldsymbol{x},t) \in \Omega_{\mathrm{NSE}} \times [0,T], \tag{1a}$$

$$\nabla\cdot\boldsymbol{u} = 0, \quad \forall(\boldsymbol{x},t) \in \Omega_{\mathrm{NSE}} \times [0,T], \tag{1b}$$

$$\boldsymbol{u} = \boldsymbol{u}^*, \quad \forall(\boldsymbol{x},t) \in (\partial\Omega_{\mathrm{NSE}} \cap \partial\Omega \setminus \Gamma_f) \times [0,T], \tag{1c}$$

$$\nabla^2\varphi = 0, \quad \forall(\boldsymbol{x},t) \in \Omega_{\mathrm{PF}} \times [0,T], \tag{1d}$$

$$\frac{\partial\varphi}{\partial n} = q^*, \quad \forall(\boldsymbol{x},t) \in (\partial\Omega_{\mathrm{PF}} \cap \partial\Omega \setminus \Gamma_f) \times [0,T], \tag{1e}$$

$$\frac{D\boldsymbol{x}}{Dt} = \boldsymbol{u}, \quad \forall(\boldsymbol{x},t) \in \Gamma_f \times [0,T], \tag{1f}$$

$$\frac{D\varphi}{Dt} = -gz + \frac{1}{2}|\nabla\varphi|^2 - \frac{p^*}{\rho} \quad \forall(\boldsymbol{x},t) \in \Gamma_f \times [0,T], \tag{1g}$$

$$\nabla\varphi = \boldsymbol{u}_{\mathrm{NSE}}, \quad \forall(\boldsymbol{x},t) \in \Gamma \times [0,T], \tag{1h}$$

$$\frac{\partial\varphi}{\partial t} + \frac{1}{2}|\nabla\varphi|^2 + gz = -\frac{p_{\mathrm{NSE}}}{\rho}, \quad \forall(\boldsymbol{x},t) \in \Gamma \times [0,T], \tag{1i}$$

where the unknown variables $(\boldsymbol{u}, p, \varphi)$ are functions of spatial and temporal coordinates $(\boldsymbol{x}, t)$, with $\boldsymbol{u}$ being velocity, p pressure and φ velocity potential. Fluid property includes density ρ and viscosity μ. Boldface symbols and superscript "$*$" indicate vectors and boundary data, respectively. For a field quantity $a, a_i = a|_i$, $i =$"NSE", "PF", and $a_\Gamma = a|_\Gamma$ denote the corresponding restriction of a. Material derivatives are denoted by $D\cdot/Dt$. In Eq. (1), (1a) and (1b) are the NSE, while Eq. (1d) is the governing equation for potential flow in Ω_{PF}. Velocity and flux boundary conditions are described in Eqs. (1c) and (1e), respectively. In our implementation, either non-slip or free-slip boundary conditions can be imposed on the walls and bottoms (see Section 5 for details). Wave is generated with a