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A parametric study of buoyancy-driven flow of two-immiscible fluids in a differentially heated inclined channel

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ABSTRACT

The buoyancy-driven interpenetration of two immiscible fluids in a differentially heated inclined channel is investigated by solving the Navier–Stokes, the continuity and the energy equations along with Cahn–Hillard equation to track the interface. After conducting a grid convergence test, a parametric study is conducted to investigate the effects of Reynolds number, Bond number, Marangoni number, density ratio, viscosity ratio and temperature difference between the walls (ΔT) on flow dynamics and interfacial instability between two immiscible fluids. We found that increasing Reynolds number, Bond number, ΔT, density ratio and decreasing viscosity ratio destabilizes the flow dynamics by increasing the intensity of vortical structures and 'mixing' of the fluids. The flow dynamics of vortical structures are altered by the imposed temperature gradient between the walls in comparison with isothermal system. We found a critical *Bo* below which the surface tension force dominates the gravitational force for the set of parameter values considered, which in turn give rise to stationary stable interface.

1. Introduction

The interpenetration of two immiscible fluids in an confined inclined channel under the action of gravitational force is frequently encounter in many industrial and natural phenomena and thus has been investigated by several authors in the past, see for instance, [1–6]. Consider two immiscible fluids having different density and viscosity occupying the upper and lower half of an inclined channel and initially separated by a partition. At time, $t = 0$, the partition is suddenly removed and the fluids are allowed to mix by the action of the gravitational force. This problem is frequently referred to as the "lock-exchange" problem [3,7–9]. This phenomenon not only plays an important role in the design of chemical and petroleum engineering processes [1,2], but also helps in understanding various natural systems in oceanography and atmospheric sciences [10].

The "lock-exchange" problem (shown in Fig. 1) has been investigated experimentally [3,11–14] and numerically [6,7] by considering fluids having equal viscosity. However, viscosity differential between fluids can have significant effect on the dynamics of the unsteady mixing process, which was recently investigated by [5]. The dimensionless parameters characterizing the flow in this problem are density contrast characterized by Atwood number, $At \equiv (\rho_B - \rho_A)/(\rho_B + \rho_A)$, the tilt angle, θ (measured from horizontal) and the viscosity ratio of the two fluids, $\mu_r(\equiv \mu_B/\mu_A)$, wherein ρ_A, μ_A and ρ_B, μ_B are the densities and viscosities of fluids 'A' and 'B', respectively. It is important to note here that all the above mentioned studies are for isothermal systems, although in most of the industrial applications this phenomena encounter with temperature gradient between the fluids and also in geometry having differentially heated boundaries. This is the subject of the present investigation. However, we shall discuss the dynamics associated with the isothermal systems first before discussing the previous works on non-isothermal systems.

In isothermal "lock-exchange" flows [14], three types of flow regimes and mixing patterns were observed depending on the values of the tilt angle. In channel with tilt angles ($90^\circ \geqslant \theta \geqslant 25^\circ$), increasing θ decreases the magnitude of the front velocity of the high and low density fluids moving in the opposite directions. The front velocity (V_f) depends on the local density contrast across the interface. In this regime, flow and mixing are influenced by two distinct processes due to the components of the gravitational force along the axial and transverse directions of the channel. The former one accelerates the two fluids into each other at comparable velocities. During this motion, the interface separating the two fluids becomes unstable giving rise to the Kelvin–Helmholtz (KH) type instabilities, and consequent transverse mixing, which in turn

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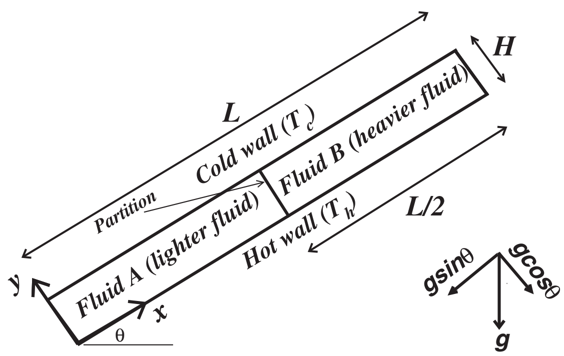

Fig. 1. Schematic diagram (not to scale) showing the initial flow configuration when the bottom wall is hotter than the top wall. A heavier fluid (fluid 'B') is overlaying a lighter one (fluid 'A') in an inclined channel. They are initially separated by a partition, which is being removed at $t = 0$. θ is the tilt angle of the channel with the horizontal. The aspect ratio of the channel is 1:40. The volume fraction of the lighter fluid, C, whose value can be taken to be 1 and 0 for the lighter and heavier fluids, respectively.

decreases the front velocity. However, later one has an opposite effect by acting to segregate the two fluids and helps to increase the front velocity. For lower tilt angles ($25^\circ > \theta > 8^\circ$) the front velocity is nearly constant, with a value approximately equals to $0.7\sqrt{Atgd}$, where g is the gravitational acceleration and d is a characteristic dimension (diameter of the pipe in the study of [14]). For near horizontal channel ($\theta < 8^\circ$) the flow transitions to a third regime where the two fluids move as counter-current Poiseuille flows; the front velocity increases with increasing the value of the tilt angle. In this regime, the flow dynamics is a result of the balance between buoyancy and wall friction. Hallez and Magnaudet [7] numerically studied the buoyancy-induced mixing of two fluids in circular, rectangular and square geometries, and found that the flow dynamics are more coherent and persistent in two than in three dimensions, which in turn give rise to more intense mixing and long-lasting flow structures in two-dimensional than in three dimensional geometries. Sahu and Vanka [6] investigated interpenetration of two immiscible fluids in a tilted channel using a lattice Boltzmann method (LBM). They conducted a parametric study by varying Atwood number, Reynolds number, tilt angle and surface tension. Their results compared well with the previous experimental results [3,13,14]. Nasr-Azadani et al. [15,16] also studied the "lock-exchange" flows in the context of turbidity currents. Next we discuss the related works conducted on non-isothermal systems.

The influence of combined buoyancy and thermal convection in immiscible liquid layers occurs in many industrial applications, such as alloying techniques, processing of ceramics and semiconductors that frequently involves molten and gaseous phases. Several authors [17–21] investigated the effects the thermal convection in multiphase flows involving immiscible fluids by considering the temperature gradient along and normal to the interface. Prakash and Koster [19] studied thermal convection of two immiscible liquids in a container, which is differentially heated along the interface. They found that the flow pattern observed in their experiment agrees well with those obtained from their theoretical calculations. Liu et al. [18] numerically studied the thermocapillary convection in a rectangular channel having temperature gradient normal to the interface of two immiscible liquids in the context of crystal growth. They found that the flow structure and temperature field are symmetric with respect to interface. It was concluded that by adjusting viscosity, conductivity of the fluids and thickness of the layers one could get desired flow patterns. Koster and Nguyen [22] showed that the appearance of two counter-rotating natural convection rolls in a system where the values of the temperature at the left and right walls lie below and above the density inversion temperature, or vice-versa. The unsteady laminar natural convection with internal heat generation in rectangular container with water as a working fluid and temperature gradient along the interface is investigated by [17]. The top and bottom walls are considered to be adiabatic. The effects of both heat generation and variations in the aspect ratio are investigated.

In spite of several studies those investigated thermal convection, to the best of our knowledge, mixing and interpenetration of two immiscible fluids in inclined channel with temperature gradient have not being investigated in literature. Also in most of the previous studies considered stable system with temperature gradient such that the lighter fluid overlays the heavier fluid. Hence the present investigation has been motivated to understand the interface deformation for unstable configuration (heavier fluid on the top of a lighter one) in an inclined channel having differentially heated walls.

The rest of the paper is organized as follows. The problem is formulated in Section 2; Section 3 presents the numerical method used in the present study. Results are presented in Section 4, where the effects of Reynolds number, temperature difference between the walls, Bond number, viscosity ratio, Marangoni number and density ratio on mixing and interfacial instability characteristics are provided. The concluding remarks are given in Section 5.

2. Formulation

We consider buoyancy-driven flow of two immiscible liquids in an inclined two-dimensional confined channel of length, L and height, H having differentially heated walls as shown in Fig. 1. The walls are considered to be rigid and impermeable. The walls at $x = (0, L)$ are insulated and the walls at $y = 0$ and $y = H$ are maintained at fixed values of temperature. Two cases are considered (i) when the bottom wall is hotter than the top wall, and (ii) the top wall is hotter than the bottom wall. T_c and T_h represent the temperature of the cold and hot walls, respectively, such that the temperature difference between the walls, $\Delta T = T_h - T_c$. The liquids are assumed to be Newtonian and incompressible. A rectangular coordinate system (x, y) is used to model the flow dynamics, where x and y denote the axial and transverse coordinates, respectively.

The flow dynamics is governed by continuity, incompressible Navier–Stokes equations along with energy equation. The diffuse interface method [4] is used to track the interface separating the immiscible fluids; the Cahn–Hillard equation for the volume fraction of the lighter fluid, C (whose value without losing generality can be taken to be 0 for the heavier fluid and 1 for the lighter fluid) is solved for this purpose.

The following scaling is employed to nondimensionalize the governing equations:

$$(x, y) = H(\tilde{x}, \tilde{y}), \quad t = \frac{H}{V}\tilde{t}, \quad (u, v) = V(\tilde{u}, \tilde{v}), \quad p = \rho V^2 \tilde{p},$$
$$\mu = \tilde{\mu}\mu_A, \quad \rho = \tilde{\rho}\rho_A, \quad \kappa = \tilde{\kappa}\kappa_A, \quad T = \widetilde{T}(T_h - T_c) + T_c, \tag{1}$$

where the tildes designate dimensionless quantities, V is the characteristic velocity, given by $\sqrt{gH}$; g being the acceleration due to gravity, μ_A, ρ_A and κ_A are the viscosity, the density and the thermal conductivity of fluid A (lighter fluid) at the reference temperature, T_c, respectively. After dropping tildes, the dimensionless governing equations are given by

$$\nabla \cdot \mathbf{u} = 0, \tag{2}$$

$$\rho\left[\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right] = -\nabla p + \frac{1}{Re}\nabla \cdot \left[\mu(\nabla \mathbf{u} + \nabla \mathbf{u}^T)\right] + F, \tag{3}$$