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Low Mach number slip flow through diverging microchannel

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This paper presents experimental and three-dimensional numerical study of gaseous slip flow through diverging microchannel. The measurements are performed for nitrogen gas flowing through microchannel with different divergence angles (4°, 8°, 12° and 16°), hydraulic diameters (118, 147 and 177 μm) and lengths (10, 20 and 30 mm). The Knudsen number falls in the continuum and slip regimes ($0.0005 \leqslant Kn \leqslant 0.1$; Mach number is between 0.03 and 0.2 for the slip regime) while the flow Reynolds number ranges between 0.4 and 1280. The static pressure drop is measured for various mass flow rates; and it is observed that the pressure drop decreases with an increase in the divergence angle. The viscous component has a relatively large contribution in the overall pressure drop. The numerical solution of the Navier–Stokes equations with the Maxwell's slip boundary condition shows absence of flow reversal (due to slip at the wall), larger viscous diffusion and lower kinetic energy in the diverging microchannel. The centerline velocity and wall shear stress decrease with an increase in the divergence angle. The numerical results further show three different flow behaviors: a nonlinear pressure variation with rapid flow deceleration in the initial part of the microchannel; uniform centerline velocity with linear pressure variation in the middle part, and flow acceleration with nonlinear pressure variation in the last part of the microchannel. A characteristic length scale for diverging microchannel is also defined. The location of the characteristic length is a function of the Knudsen number and shifts toward the microchannel inlet with rarefaction. Mass flow rate and pressure distribution along the channel are also obtained numerically from the direct simulation Monte Carlo (DSMC) method and compared suitably with the experimental data or Navier–Stokes solutions. Empirical relations for the mass flow rate and Poiseuille number are suggested. These results on gaseous slip flow through diverging microchannels are considerably different than their continuum counterparts, and are not previously available.

1. Introduction

Expansion, which may be sudden or gradual, appears fundamental to micro-systems such as microactuators, microturbines, gas chromatographs, and micro air vehicles. Therefore, study of rarefied gas flow through diverging microchannel is important for both engineering and scientific applications. It is important to understand the gas flow behavior through such geometry in terms of pressure drop, velocity distribution and flow structure. Most of the earlier work however focused on uniform cross-section microchannels with liquid or gas flow [1–12] and flow through other cross-section area microchannels have been inadequately studied. The low pressure gas flows through conventional tubes are governed by the same set of non-dimensional parameters as that of gas microflow; Sreekanth [13] and Demsis et al. [14,15]. A comprehensive review on gas flow in microchannel was reported by Agrawal [16].

Relatively few studies explore *incompressible* flow through microchannel with expansion. Pan et al. [17] observed flow separation at the junction for liquid flow through a microchannel with a sudden expansion. Similarly, the experimental study on liquid flow by Tsai et al. [18] indicated formation of a separation vortex at the sudden expansion corner of a high aspect ratio microchannel. Duryodhan et al. [19] inferred that flow separation occurs in a diverging microchannel with water flow beyond a critical divergence angle of 16°; the critical value of divergence angle agrees with the corresponding value for continuum flow. Abdelall et al. [20] experimentally investigated the pressure drop caused by flow of

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(continuum) air and water through sudden expansion in a microtube ($1000 < Re < 7000$). They reported that the expansion and contraction loss coefficients are different for air and water flows. The above studies indicate that flow separation occurs with liquid flow through microchannel with sudden/gradual expansion.

Study of liquid flow through diverging/converging sections has also been undertaken in the context of micro-pump by Stemme and Stemme [21], Olson et al. [22], Gerlach [23], Singhal et al. [24] and Wang et al. [25]. Akbari et al. [26] studied liquid flow through a series of diverging microchannels and proposed an analytical model for calculating the flow resistance.

The behavior of *rarefied gas flow* is however different than that of the liquid flow as reviewed next. Rathakrishnan and Sreekanth [27] studied rarefied gas flow (Kn = 0.0026–1.75, Nitrogen) through circular tube with sudden increase in cross sectional area. They noted that in the transition regime, the pressure ratio and length to diameter ratio of the passage strongly influence the discharge through sudden enlargements. In a recent experimental study involving flow through a sudden expansion, Varade et al. [28] observed a discontinuity in the slope of pressure and absence of flow separation at the junction; in the slip regime. These measurements are qualitatively similar to the two-dimensional planar simulations of Agrawal et al. [29]. Lee et al. [30] in an experimental study on gas flow through microchannels connected through diverging section observed that the mass flow rate decreases and the pressure loss increases with increasing included angle of the transition section.

It can be noted that a systematic and detailed analysis for rarefied gas flow through diverging microchannels is not available. Further, there is ambiguity in the literature about the characteristic dimension to be employed for obtaining non-dimensional parameters [21,23,31]. These issues provided the motivation for undertaking the current work. The objectives of this work are to investigate the gas flow behavior through a diverging microchannel and to highlight significant differences with respect to continuum flow behavior. The static pressure drop is measured for different mass flow rates and analyzed in this work. The numerical simulations are performed to obtain the local variation in flow parameters for gaseous slip flow through diverging microchannel.

2. Measurement system

2.1. Experimental facility

The experimental facility is similar to that employed by Varade et al. [28,45,46]. It consists of a vacuum system, inlet reservoir, outlet reservoir and mass flow controller as shown in Fig. 1a. The vacuum system consists of a diffusion pump (maximum pumping speed of 700 lpm, 1.31×10^{-2} kg/s) and a rotary pump (speed of 350 lpm, 6.55×10^{-3} kg/s). The lowest absolute pressure that can be accomplished by the vacuum system is 10^{-4} Pa. An air filter is mounted in the incoming gas stream for blocking particles (particle size > 25 μm). The different Reynolds and Knudsen numbers are attained by using two different mass flow controllers (M/s MKS Instruments, range 0–20 and 0–200 sccm or standard cubic centimeters per minute). The absolute pressure at the test section is measured by two absolute pressure transducers (also from M/s MKS Instruments, range 0–10,000 or 0–100,000 Pa). The uncertainty in measurement of flow rate and pressure along with other measured and derived parameters is provided in Table 1.

2.2. Diverging microchannel

The diverging microchannel with trapezoidal cross section geometry is shown in Fig. 1b. The gas flow openings at the inlet and outlet reservoir are provided with a 'T' section tap each. The first prong of the 'T' section is fixed in the reservoir opening; the second prong is used for nitrogen gas flow whereas the remaining prong serves the measurement of absolute pressure through a microtube. The microchannel is fabricated at the Centre of Excellence in Nanoelectronics (CEN) at IIT Bombay. A silicon wafer (100, p-type) with one side polished is used for microchannel fabrication. The microchannel fabrication process starts with wafer cleaning and surface preparation and photo mask preparation followed by positive photo resist coating, prebaking, UV radiation exposure, post exposure baking, developing, wet etching and finally geometry characterization using profilometer. The surface roughness of approximately 0.1 μm is observed for all the microchannels. The microchannel is sealed with a quartz plate using PDMS (Polydimethylsiloxane) bonding. The geometrical parameters of all microchannels are documented in Tables 2–4.

2.3. Experimental procedure

The validation of the experimental set up is performed by measuring the pressure drop for different mass flow rates through a straight microtube (internal diameter of 800 μm). The experimental fRe value is compared against the fRe value obtained from the correlation of Verma et al. [8]

$$fRe = \frac{64}{1 + 14.88Kn} \tag{1}$$

where f = Darcy friction factor. The above correlation was formulated using experimental data from several researchers and is applicable for laminar flow in a smooth circular tube. The deviation in the experimental fRe value was observed to be less than 4% of the value obtained from the correlation; thereby the experimental setup and data reduction procedure is considered to be validated.

Thereafter, experiments for diverging microchannel are carried out. The setup leakage testing is performed using the procedure reported in Demsis et al. [15]. The leakage is ensured to be less than 2% of the mass flow rate employed in the measurements. The uncertainty in mass flow rate consists of the combined uncertainty in the measurement and leakages. The absolute static pressure is measured at the inlet and outlet of the diverging microchannel for different mass flow rates of nitrogen at 300 K. The temperature of the gas at inlet and outlet has been explicitly measured (Table 5). The maximum variation in temperature between the inlet and outlet was noted to be 0.8 K for the slip flow and 1.7 K for the continuum flow. Additional confirmation of the temperature variation is obtained through numerical simulation (results presented later in Section 4.1); which indicates that the flow is nearly isothermal. The flow Reynolds number is between 0.4 and 1280 and the Knudsen number is between 0.0005 and 0.10; thereby covering the continuum and slip regimes. The Mach number is between 0.03 and 0.2 for the slip regime. The ranges of test conditions employed in the measurements are tabulated in Tables 2–4.

2.4. Data reduction

The friction factor f is estimated using following equation [32] which takes into account both viscous and acceleration/deceleration effects:

$$\Delta P = \frac{G^2}{2}\left[\frac{fL}{D\rho_m} + 2\left(\frac{1}{\rho_o} - \frac{1}{\rho_i}\right)\right] \tag{2}$$

Here ΔP = pressure drop, Pa; f = Darcy friction factor; L = length of the tube, m; D = hydraulic diameter, m; ρ_m = mean density, ρ_o = density at outlet, ρ_i = density at inlet, kg/m^3. Using the ideal gas