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Application of the HAM-based Mathematica package BVPh 2.0 on MHD Falkner–Skan flow of nano-fluid

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ABSTRACT

Many boundary-layer flows are governed by one or coupled nonlinear ordinary differential equations (ODEs). Currently, a Mathematica package BVPh 2.0 is issued for nonlinear boundary-value/eigenvalue problems with boundary conditions at multiple points. The BVPh 2.0 is based on an analytic approximation method for highly nonlinear problems, namely the homotopy analysis method (HAM), and is free available online. In this paper, the BVPh 2.0 is successfully applied to solve magnetohydrodynamic (MHD) Falkner–Skan flow of nano-fluid past a fixed wedge in a semi-infinite domain, and the influence of physical parameters on the considered flows is investigated in details. Physically, this work deepens and enriches our understandings about the magnetohydrodynamic Falkner–Skan flows of nano-fluid past a wedge. Mathematically, it illustrates the potential and validity of the BVPh 2.0 for complicated boundary-layer flows.

1. Introduction

In fluid mechanics, the Falkner–Skan flow is significant and fundamental in both theory and practice. In particular, such kind of flows occur frequently in enhanced oil recovery, packed bed reactor geothermal industries, etc. Moreover, there is growing interest of the researchers in the magnetohydrodynamics (MHD) flows, mainly due to their vast applications in power generators, design of heat exchangers, electrostatic filters, the cooling of reactor, MHD accelerators and so on. The magnetic field has also stabilizing effects in this kind of boundary layer flows. Hence, many researchers have been doing their contributions to the Falkner–Skan flows with or without a magnetic field. For example, Abbasbandy and Hayat [1,2] analyzed the MHD Falkner–Skan flow of viscous fluid using the homotopy analysis and Hankel–Pade methods. Parand et al. [3] developed approximate solution of MHD Falkner–Skan flow by the Hermite functions pseudo-spectral method. The solutions of reversed flow of the Falkner–Skan equations were obtained by Yang and Lan [4]. Yao [5,6] examined the uniform suction and heat transfer influences in the Falkner–Skan wedge flow. The generalized Falkner–Skan equations in the FENE fluid was investigated by Anabtawi and Khuri [7]. The numerical solutions of the Falkner–Skan equations in viscous fluid were gained by Zhu et al. [8]. The Falkner–Skan flow due to stretching surface was addressed by Yao and Chen [9]. Alizadeh et al. [10] obtained the solutions of Falkner–Skan equation with wedge using the Adomian decomposition method.

Nano-particles are objects with at least one dimension smaller than 100 nm (preferably <10 nm), where a nanometer (nm) is one-billionth of a meter. Although nano-particles are so small, they often possess far more remarkable characteristics than the same material and bulk without them. Fluids such as oil, water and ethylene glycol mixtures are naturally poor in heat transfer. In the past decades, lots of researches were done to develop fluids with ultrahigh-performance such as the enhanced electrical conductivity, intensified heat transfer, improved oils, coolants and industrial equipments. When a very small amount of nano-particles are dispersed uniformly and suspended stably in clear fluids, the thermal properties of the fluid changes significantly. Choi [11] called these fluids as nano-fluids, and proposed that nanometer-sized metallic particles can be suspended in industrial heat transfer fluids. Therefore, a nano-fluid is a suspension of nano-particles in a traditional base fluid, which enhances the heat transfer characteristics of the

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clear fluid. Nano-technology has applications in automatize industry, electronic devices, such as supercomputers, cooling systems, power plants, and artificial organs. Choi et al. [12] observed that the thermal conductivity of fluid can be increased up to approximately two times by means of adding a small amount (less then 1%) of nano-particles to clear liquids. For reviews on nano-fluids, please refer to Das et al. [13] and Wang and Mujundar [14]. Recently, the Falkner–Skan problem for a static or moving wedge in hydrodynamic viscous nano-fluids has been numerically investigated by Yacob et al. [15] and Fallah et al. [16]. They gained the Pareto optimal solutions by means of multi-objective genetics algorithms. Khan et al. [17] numerically examined the Falkner–Skan boundary layer flow of nano-fluid over wedge with convective boundary condition. Khan and Pop [18] considered steady boundary-layer flows past a stretching wedge in a viscous nano-fluid with a parallel free stream velocity $u_e(x)$.

In this paper, a steady-state laminar two dimensional MHD boundary layer flow of viscous nano-fluid past a fixed wedge is considered. The magneto nano-fluids are important in applications related to modulators, optical switches, optical gratings, tunable optical fiber filters and so on. The magnetic nano-particles are important in medicine, sink float separation, cancer therapy, magnetic cell separation, construction of loud speakers, magnetic resonance imaging and so on. In the form of the boundary-layer theory, the considered problem is governed by three coupled nonlinear ordinary differential equations (ODEs) in a semi-infinite domain with boundary conditions at infinity. Traditionally, such kind of coupled nonlinear ODEs can be solved by means of numerical methods such as the finite difference method (FDM) by means of moving the boundary condition at infinity to a finite but far enough position which causes some uncertainty and inaccuracy to its numerical solutions. Unlike the traditional approaches, we use here a Mathematica package BVPh 2.0 [19] for nonlinear boundary-value/eigenvalue problems governed by coupled nonlinear ODEs with multiple boundary conditions, which can exactly satisfy the boundary condition at infinity, since the BVPh 2.0 is based on the computer algebra system Mathematica and makes computations with functions instead of numbers. Mathematically, the BVPh 2.0 is based on the homotopy analysis method (HAM) [20–23], an analytic approximation technique for highly nonlinear problems and has many advantages compared to the traditional ones. First, based on the homotopy in topology, the HAM can always transfer a nonlinear problem into an infinite number of linear sub-problems without any small/large physical parameters, and besides provides us great freedom to choose the equation-type and base function of solution of these linear sub-problems for high-order approximations. Especially, unlike all other analytic approximation methods, the HAM provides us a simple way to guarantee the convergence of solution series, so that it is valid for problems with high nonlinearity. The general validity and power of the HAM have been illustrated by hundreds of successful applications of the HAM in various fields of science, finance and engineering. To simplify the applications of the HAM, some HAM-based packages in Mathematica or Maple have been developed. The BVPh 2.0 is one of them, which is an easy-to-use tool for boundary-layer flows, and is free available online (http://numericaltank.sjtu.edu.cn/BVPh.htm).

In this paper, the BVPh 2.0 is used to solve the considered magnetohydrodynamics (MHD) Falkner–Skan flow of nano-fluid. The influence of physical parameters on the profiles of velocity, temperature and concentration, the local skin friction coefficient, the local Nusselt number and the local Sherwood number is investigated in details. Physically, this work deepens and enriches our understandings about the magnetohydrodynamics (MHD) Falkner–Skan flow of nano-fluid. Mathematically, it illustrates that the HAM-based Mathematica package is indeed an easy-to-use tool for complicated boundary-layer flows.

2. Mathematical formulas

We consider here a steady-state laminar incompressible two dimensional boundary-layer flow of nano-fluid past a fixed wedge. A constant magnetic field of strength B acts in a transverse direction to flow. The fluid is electrically conducting in the presence of applied magnetic field B. The induced magnetic field effect is not taken into account. Such consideration holds when magnetic Reynolds number is chosen small. In addition the influence of electric field is negligible. In this situation the current density becomes $\bar{J} = \sigma(\overline{V} \times \overline{B})$ and lorentz force $\bar{J} \times \overline{B} = -\sigma B^2 V$. In-fact zero electric field corresponds to the case when polarization effects are not considered. The Hall effect is also not taken into account. Let T_w, C_w and T_∞, C_∞ denote the temperature and concentration at the surface and at infinity, respectively. Effects of Brownian motion and thermophoresis are considered. Under these assumptions, the governing equations of continuity, momentum, energy and nano-particle volume fraction are as follows (see Fig. 1):

(i) Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}$$

(ii) Equation of motion

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho_f}(u - U). \tag{2}$$

(iii) Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left\{D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\}. \tag{3}$$

(iv) Nanoparticle volume fraction equation

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial y^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

where u, v are the x- and y-components of the fluid velocity, U denotes the inherent characteristic velocity, T the temperature, α the thermal diffusivity, τ the ratio of heat capacity of nano-particle to that of the base fluid, ν the kinematic viscosity, σ the electrical conductivity, K the thermal conductivity, T_∞ and C_∞ the free stream temperature and concentration, C the nano-particle volume fraction, D_B the Brownian diffusion coefficient, D_T the thermophoretic diffusion coefficient and ρ_f the fluid density, respectively. The corresponding initial and boundary conditions are

$$u = 0, \quad v = 0, \quad T = T_w, \quad C = C_w(x) \quad \text{at} \quad y = 0, \tag{5}$$

$$u = U(x) = ax^n, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty. \tag{6}$$

with

$$B = B_0 x^{(n-1)/2}, \tag{7}$$

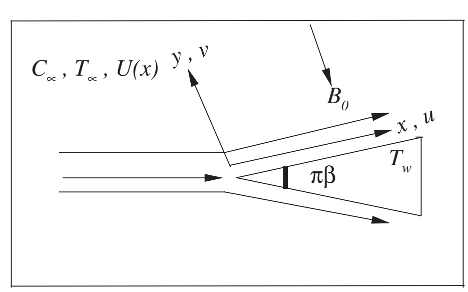

Fig. 1. Physical configuration.