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Internal natural convection driven by an orthogonal pair of differentially heated plates

P. Kandaswamy, A.K. Abdul Hakeem [1], S. Saravanan *

UGC-DRS Center for Fluid Dynamics, Department of Mathematics, Bharathiar University, Coimbatore 641 046, Tamil Nadu, India

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ABSTRACT

An internal natural convective flow arising in a closed square cavity due to the presence of two orthogonal and differentially heated plates is studied numerically. The flow is assumed to be laminar and two-dimensional. The Alternating Direction Implicit technique in association with the Successive Over Relaxation method is used to solve the coupled nonlinear governing equations. The heat and fluid flow interactions are studied for various combinations of plate–plate and plate–wall temperature ratios and different positions of the heated plates. When there are two plates with high temperature contrast the heat transfer mechanism inside the cavity is mainly ruled by the hotter one. In general convection heat transfer becomes strengthened when the vertical plate is hotter.

1. Introduction

The fast growth in electronic technology has increased component densities and power dissipation tremendously and has really become a challenge to an effective cooling. Cooling of electronic equipment packages by means of natural convection has been accepted as a viable alternative to forced cooling in some circumstances [1–3]. The optimal thermal design of electronic devices depends on an accurate choice of not only geometrical configuration but also heat source distribution to promote thermo-circulation flow rate that minimizes the rise in temperature. Natural convective flows play an important role in many other engineering applications also like nuclear design, solar energy collection, space heating, etc. There have been plenty of experimental and theoretical works belonging to the past few decades investigating the mechanism of natural convection inside enclosures with various boundary conditions and geometries [4–6]. Many studies (see [7–11]) deal with the influence of baffles/thin plates/discretely heated elements when they are wall mounted.

The problem of natural convection in enclosures containing localized heat sources inside is of practical concern. Oztop et al. [12] have addressed such an issue with a thin heated plate built-in vertically or horizontally and found that heat transfer can be enhanced by 20% when the plate is located vertically. Boukhattem et al. [13] have studied the same problem recently with different boundary conditions. Natural convection in a closed enclosure with two heated vertical plates forming an internal channel placed at the center was numerically examined by Barozzi et al. [14]. They considered the plates to be of either isothermally heated or of uniformly heat generating. Lee and Ha [15] investigated the problem of natural convection in a square enclosure with a heat-generating conducting body which is placed at the center of the cavity. Natural convection in a square enclosure containing several disconnected conducting solid blocks was analyzed numerically by Merrikh and Lage [16]. Recent studies on localized heating in closed cavities due to the presence of hot thin plates inside have taken into account complex factors like the presence of nanoparticles, inclination, forced convection, surface radiation, etc. (see [17–21]). Heat transfer in enclosures with two mutually perpendicular heated plates has received limited attention though such configurations are encountered in microelectronics industry. Papanicolaou and Jaluria [22] and Icoz and Jaluria [23] have considered such configuration in their design and optimization of cooling systems for electronic equipments. Natural convection in a square cavity induced by two mutually perpendicular heated plates were studied by the authors [24,25]. Mixed convection in a lid-driven enclosure filled with a nanofluid and induced by two mutually orthogonal thin plates have been investigated by Wang et al. [26].

An exhaustive literature review on internal natural convection uncovered only very few studies dealing with the interaction among distinct heat sources of their own strengths. Such situations

* Corresponding author.
E-mail address: sshravan@lycos.com (S. Saravanan).

[1] Present address: Department of Mathematics, Sri Ramakrishna Mission Vidyalaya College of Arts and Science, Coimbatore 641 020, Tamil Nadu, India.

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often occur in electro-optical devices wherein optical components have a lower maximum operating temperature than the electrical components [27]. Depending on their placement electronic components may raise the operating temperatures of the optical components above the allowable maximum and reduce the amount of power the optical component can dissipate without overheating. Deng et al. [28] have investigated the interaction between two discrete heat sources of different thermal strengths and found that the weaker source is always located in the wakes of the stronger one, and hence the heat is accumulated towards the stronger source. Weinstein et al. [29] experimentally studied natural convection and passive heat rejection from two heat sources maintained at different temperatures on a single circuit board. They investigated the maximum power dissipation in terms of the heat source location. Saravanan and Sivaraj [30] have recently studied natural convection in an enclosure with two vertical heat generating baffles of different strengths. It was found that the blocking effect of the baffles strongly depends on heat generation ratio and spacing between them. In the present work we study natural convection in a square cavity with two mutually perpendicular heated plates of different temperatures which are encountered frequently in electronic industry. We hope that this fundamental theoretical study would provide some support in improving the design and fabrication of fully closed electronic equipment chambers.

2. Problem formulation and solution procedure

The schematic diagram of a two-dimensional square cavity of length L is shown in Fig. 1. It is a square cavity containing a vertical plate and a horizontal plate maintained at temperatures θ_{h_1} and θ_{h_2} respectively. The plates are treated as line sources which are placed parallel to the walls of the cavity. The horizontal and vertical plates are at distances d_1 and d_2 from the center O of the cavity respectively. When the two plates intersect the temperature at the point of intersection is $\theta_a = (\theta_{h_1} + \theta_{h_2})/2$. This can be justified when both the plates have identical material properties. In such a situation a linear variation of the temperature of the plate is assumed from the points of intersection to the respective end points. The vertical and horizontal walls are isothermally maintained at a constant temperature θ_c, which is lower than those of the heated plates. The Cartesian co-ordinates (x_1, x_2) with the corresponding velocity components (v_1, v_2) are chosen. The gravity g acts downwards parallel to the x_1 direction.

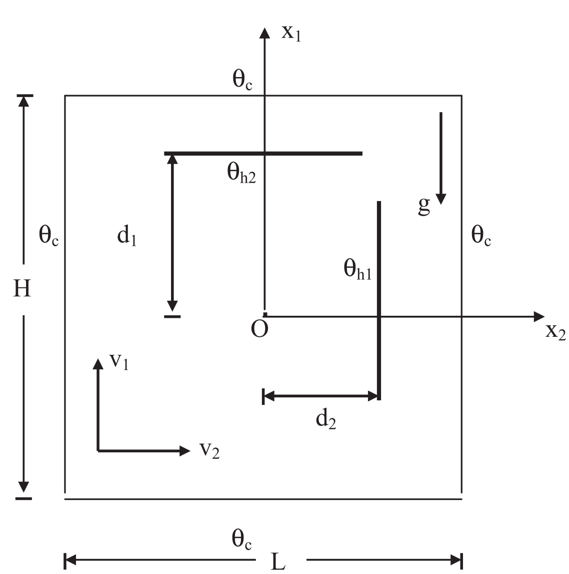

Fig. 1. Physical configuration.

The equations governing the motion of an incompressible flow of the fluid under Oberbeck–Boussinesq approximation in an environment described above are

$$\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} = 0 \tag{1}$$

$$\frac{\partial v_1}{\partial t} + v_1\frac{\partial v_1}{\partial x_1} + v_2\frac{\partial v_1}{\partial x_2} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x_1} + \nu\nabla^2 v_1 - g\beta(\theta - \theta_c) \tag{2}$$

$$\frac{\partial v_2}{\partial t} + v_1\frac{\partial v_2}{\partial x_1} + v_2\frac{\partial v_2}{\partial x_2} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x_2} + \nu\nabla^2 v_2 \tag{3}$$

$$\frac{\partial \theta}{\partial t} + v_1\frac{\partial \theta}{\partial x_1} + v_2\frac{\partial \theta}{\partial x_2} = \alpha\nabla^2\theta \tag{4}$$

where α is the thermal diffusivity, β the volumetric expansion coefficient, θ the temperature, μ the dynamic viscosity, ν the kinematic viscosity and ρ the density of fluid. The appropriate initial and boundary conditions are

$$t = 0: \quad v_i = 0; \quad \theta = \theta_c; \quad \text{at} \quad -\frac{L}{2} \leqslant x_i \leqslant \frac{L}{2} \tag{5}$$

$$t > 0: \quad v_i = 0; \quad \theta = \theta_c; \quad \text{at} \quad x_i = \pm\frac{L}{2} \tag{6}$$

$$v_i = 0 \quad \text{on the plates}$$

$$\theta = \begin{cases} \theta_{h_1} & \text{on the vertical plate} \\ \theta_{h_2} & \text{on the horizontal plate} \end{cases} \tag{7}$$

When the plates intersect

$$\theta = \begin{cases} \theta_{h_1} + (\theta_a - \theta_{h_1})\left(\dfrac{x_1 + L/4}{d_1 + L/4}\right) & x_1 \in [-L/4, d_1] \ \text{on the vertical plate} \\ \theta_a + (\theta_{h_1} - \theta_a)\left(\dfrac{x_1 - d_1}{L/4 - d_1}\right) & x_1 \in [d_1, L/4] \end{cases}$$

$$\theta = \begin{cases} \theta_{h_2} + (\theta_a - \theta_{h_2})\left(\dfrac{x_2 + L/4}{L/4 + d_2}\right) & x_2 \in [-L/4, d_2] \ \text{on the horizontal plate} \\ \theta_a + (\theta_{h_2} - \theta_a)\left(\dfrac{x_2 - d_2}{L/4 - d_2}\right) & x_2 \in [d_2, L/4] \end{cases}$$

The vorticity-stream function $(\zeta - \psi)$ formulation of Eqs. (1)–(4) after non-dimensionalization can be written as

$$\frac{\partial \zeta}{\partial \tau} + J(\psi, \zeta) = Gr\frac{\partial T}{\partial X_2} + \nabla^2\zeta \tag{8}$$

$$\frac{\partial T}{\partial \tau} + J(\psi, T) = \frac{1}{Pr}\nabla^2 T \tag{9}$$

$$\nabla^2\psi = -\zeta \tag{10}$$

where $J(\psi, \zeta)$ is the Jacobian of ψ, ζ with respect to X_1, X_2, $Gr = g\beta(\theta_{h_j} - \theta_c)L^3/\nu^2$ the Grashof number and $Pr = \nu/\alpha$ the Prandtl number. The nondimensional variables used in the above equations are

$$X_i = \frac{x_i}{L},\ D_i = \frac{d_i}{L},\ \tau = \frac{t}{L^2/\nu},\ T = \frac{\theta - \theta_c}{\theta_{h_j} - \theta_c},\ \Psi = \frac{\psi}{\nu}\ \&\ \zeta = \frac{\omega}{\nu/L^2} \tag{11}$$

The subscript $_j$ is fixed as 1 if the horizontal plate is hotter and 2 if the vertical plate is hotter. Unlike in [24,25] we have plate–plate and plate–wall temperature ratios. We introduce them through the non-dimensional temperature ratios $\Theta_1 = \theta_{h_1}/\theta_{h_j}$, $\Theta_2 = \theta_{h_2}/\theta_{h_j}$ and $\Theta_3 = \theta_c/\theta_{h_j}$ to study the effect of various combinations of imposed temperatures. We find that Θ_3 is meaningless when $\Theta_1 = \Theta_2$. It is also important to note at this stage that quantitative comparison of the fluid flow characteristics can be made only when