Computers & Fluids 104 (2014) 9–19

Contents lists available at ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

A finite volume method for numerical simulation of shallow water models with porosity

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ARTICLE INFO

Article history:
Received 7 September 2013
Received in revised form 7 June 2014
Accepted 21 July 2014
Available online 4 August 2014

Keywords:
Shallow water equations
Finite volume method
Source terms
Porosity
Unstructured grids

ABSTRACT

In the current work, we present a new finite volume method for numerical simulation of one and two dimensions of shallow water equations with porosity. The introduction of a porosity into shallow water equations leads to modified expressions for the fluxes and source terms. The proposed method consists of two stages, which can be viewed as a predictor–corrector procedure. The first stage (predictor) of the scheme depends on a local parameter allowing to control diffusion, which modulate by using the limiters theory. The second stage (corrector) recovers the conservation equation. Numerical results are presented for shallow water equations with porosity. It is found that the proposed finite volume method offers a robust and accurate approach for solving shallow water equations with source term and porosity.

1. Introduction

The high performance computer helped simulation of complicated urbanized and marine environment problems about properties and behavior of fluid. The control of properties and behavior of fluid flow and relative parameters of the inclusive soluble materials are of great advantages offered by numerical simulation of fluid flow problems. Hence, developing efficient and accurate numerical algorithms suitable for complex flow domain has become a challenging task. The interest for flood simulation in urban area has recently led to research with addition of the porosity in the shallow water models [19]. In that sense, porosity can be used to represent the effect that area subject to flooding is only a fraction of the total surface area [8]. The difficulty of shallow water system of equations is the preservation of nontrivial equilibrium due to the presence of source terms. In the last years many authors treated this question along with an early idea of Roe [17] to upwind the source terms at the interfaces, see for instance [3,9]. In [16], the authors proposed a finite volume Roe solver for two-dimensional Euler equations with porosity, and an HLLC – a modified Riemann solver for two-dimensional shallow water equations with porosity [10]. A new AS solver, named PorAS, to solve hyperbolic system of conservation laws with porosity involving source terms, introduced in [8]. In [25], the authors used finite volume method and Roe-type approximate Riemann solver to solve one dimensional shallow water equations with porosity. In [24], the modified Roe-Type approximate Riemann solver for numerical solution of shallow water equations with porosity on unstructured grids was used. The source terms of the bed slope and porosity are both decomposed in the characteristic direction so that the numerical scheme can exactly satisfy the conservative property. In [20], the authors contrast porosity in the context of storage versus conveyance, but made no attempt to calculate these parameters individually based on cell or edge based topographic or building features. Instead, a spatially uniform and isotropic porosity was used to model an urban zone, also in [6], the authors compared two different numerical discretizations for the two-dimensional shallow water equations with porosity, both of them are high-order scheme. The numerical schemes proposed are well-balanced, in the sense that they preserve naturally the exact hydrostatic solution without the need of high-order corrections in the source terms. The aim of the present work is to implement a robust algorithm for solving shallow water equations with porosity in one and two dimensions. The emphasis is given to a modified Rusanov method studied and analyzed in [12,13] for the spatial discretization. This method is simple easy to implement, accurate and moreover it avoids the solution of Riemann problems during the time integration process. The combined method is linearly stable provided the condition for the canonical Courant–Friedrichs–Lewy (CFL) is satisfied. The rest of this paper is organized as follows: In Section 2, we introduce the shallow water equations with porosity in one-dimension. In Section 3, we present the finite volume

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http://dx.doi.org/10.1016/j.compfluid.2014.07.020
0045-7930/

method for shallow water equations with porosity, Section 3 also includes the reconstruction of the numerical fluxes in the finite volume discretization. In Section 4, we introduce the two dimensional shallow water equations with porosity and the construction of finite volume scheme in two-dimension. In Section 5, we introduce the treatment of the source term. In Section 6, we present numerical results for shallow water equations with porosity. Section 8 summarizes the results of this paper with concluding remarks.

2. One-dimensional shallow water equations with porosity

The shallow water equations with porosity in one dimension can be written in conservation form as follows:

$$\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{W})}{\partial x} = \mathbf{Q}(\mathbf{W}), \tag{1}$$

where $\mathbf{W}$ is the conserved variable, $\mathbf{F}(\mathbf{W})$ is the physical flux and $\mathbf{Q}(\mathbf{W})$ is the source term. They are given by

$$\mathbf{W} = \begin{pmatrix} \phi h \\ \phi hu \\ \phi \end{pmatrix}, \qquad \mathbf{F}(\mathbf{W}) = \begin{pmatrix} \phi hu \\ \phi hu^2 + \frac{1}{2} g\phi h^2 \\ 0 \end{pmatrix},$$
$$\mathbf{Q}(\mathbf{W}) = \begin{pmatrix} 0 \\ S_0 + S_f + \frac{1}{2} gh^2 \frac{\partial \phi}{\partial x} \\ 0 \end{pmatrix}, \tag{2}$$

where h is the water depth, u is the water velocity, ϕ is the porosity, g is the acceleration due to gravity. S_0 and S_f are the source terms corresponding to the bottom slope and the friction respectively, defined as

$$S_0 = -gh\phi \frac{\partial Z}{\partial x}, \quad S_f = gh\phi \frac{u^2}{K^2 h^{\frac{4}{3}}},$$

where Z is the bottom elevation and K is the Strickler coefficient. Here, we neglect S_f. The eigenvalues of the Jacobian matrix

$$A(W) = \frac{\partial \mathbf{F}(\mathbf{W})}{\partial \mathbf{W}} = \begin{pmatrix} 0 & 1 & 0 \\ c^2 - u^2 & 2u & -\frac{1}{2}c^2 \\ 0 & 0 & 0 \end{pmatrix},$$

are

$$\lambda_1 = 0, \quad \lambda_2 = u - c, \quad \lambda_3 = u + c, \tag{3}$$

where $c = \sqrt{gh}$.

If we write the system (1) as follows

$$\frac{\partial \mathbf{W}}{\partial t} + A(\mathbf{W}) \frac{\partial (\mathbf{W})}{\partial x} = \mathbf{Q}(\mathbf{W}), \tag{4}$$

it becomes non conservative system, there are many papers focused on the study of nonconservative hyperbolic system, see for example [7,15,5]. For the system (1), it is well-known that the non conservative term induced by the porosity variation $\frac{1}{2}gh^2 \frac{\partial \phi}{\partial x}$ and source term $-gh\phi \frac{\partial Z}{\partial x}$ leads to mathematical and numerical difficulties [21].

In our work, we use the discretization of the source term and porosity variation term to satisfies the C-property, see Sections 3 and 5.

3. Well-balanced modified Rusanov methods

In order to formulate our finite volume method, we discretize the spatial domain into control volume $\Delta x = x_{i+1/2} - x_{i-1/2}$ and we divide the temporal domain into subintervals $[t_n, t_{n+1}]$ with uniform size Δt. Following the standard finite volume formulation, we integrate the considered Eq. (1) with respect to time and space over the domain $[t_n, t_{n+1}] \times [x_{i-1/2}, x_{i+1/2}]$ to obtain the following discrete equation

$$W_i^{n+1} = W_i^n - \frac{\Delta t}{\Delta x}\left(F\left(W_{i+1/2}^n\right) - F\left(W_{i-1/2}^n\right)\right) + \Delta t Q_i^n, \tag{5}$$

where W_i^n is the time–space average of the solution W in the domain $[x_{i-1/2}, x_{i+1/2}]$ at time t_n *i.e.*

$$W_i^n = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} W(t_n, x)dx,$$

and $F\left(W_{i\pm1/2}^n\right)$ is the numerical flux at $x = x_{i\pm1/2}$ and time t_n. The spatial discretization of Eq. (5) is complete when a numerical construction of the fluxes $F\left(W_{i\pm1/2}^n\right)$ is chosen and a discretization of the source term Q_i^n is performed. In general, the construction of the numerical fluxes $F\left(W_{i\pm1/2}^n\right)$ in the finite volume discretization (5) requires a solution of Riemann problems at the cell interfaces $x_{i\pm1/2}$. Let us assume that the self-similar solution to the Riemann problem associated with Eq. (1) subject to the initial condition

$$W(x, 0) = \begin{cases} W_L, & \text{if } x < 0, \\ W_R, & \text{if } x > 0, \end{cases} \tag{6}$$

is given by

$$W(t, x) = R_s\left(\frac{x}{t}, W_L, W_R\right),$$

where R_s is the Riemann solution which has to be either calculated exactly or approximated. Thus, the intermediate state $W_{i+1/2}^n$ in (5) at the cell interface $x = x_{i+1/2}$ is defined as

$$W_{i+1/2}^n = R_s\left(0, W_i^n, W_{i+1}^n\right). \tag{7}$$

From a computational viewpoint, this procedure is very demanding and may restricts the application of the method for which Riemann solutions are difficult to approximate or simply are not available. In order to avoid these numerical difficulties and reconstruct an approximation of $W_{i+1/2}^n$, we adapt a modified Rusanov method proposed in [12,4,14] for numerical solution of conservation laws with source terms. The central idea is to integrate Eq. (1) over a control domain $[t_n, t_n + \theta_{i+1/2}^n] \times [x^-, x^+]$ containing the point $(t_n, x_{i+1/2})$ as depicted in Fig. 1. Notice that, the integration of Eq. (1) over the control domain $\left[t_n, t_n + \theta_{i+1/2}^n\right] \times [x^-, x^+]$ is used only at a predictor stage to construct the intermediate states $W_{i\pm1/2}^n$ which will be used in the corrector stage (5). Here, $U_{i\pm1/2}^n$ can be viewed as an approximation of the averaged Riemann solution R_s over the control volume $[x^-, x^+]$ at time $t_n + \theta_{i+1/2}^n$. Thus, the resulting intermediate state is given by

$$\int_{x^-}^{x^+} W\left(t_n + \theta_{i+1/2}^n, x\right)dx = \Delta x^- W_i^n + \Delta x^+ W_{i+1}^n - \theta_{i+1/2}^n\left(F(W_{i+1}^n) - F(W_i^n)\right) + \theta_{i+1/2}^n(\Delta x^- - \Delta x^+)Q_{i+\frac{1}{2}}^n, \tag{8}$$

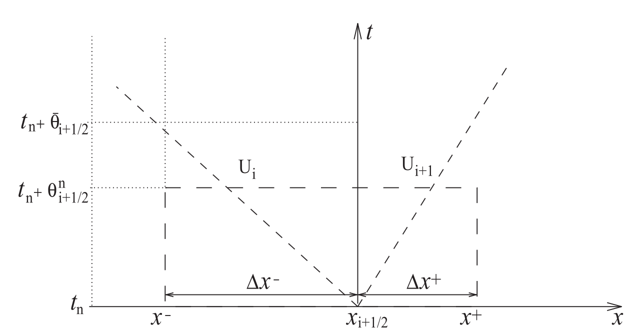

Fig. 1. The control space–time domain in the modified Rusanov method.