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Large eddy simulation on curvilinear meshes using seventh-order dissipative compact scheme

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ABSTRACT

This work investigates the performance of the high-order implicit large eddy simulation (HILES) on curvilinear meshes. The HILES is developed based on a seventh-order hybrid cell-edge and cell-node dissipative compact scheme (HDCS-E8T7) satisfying the surface conservation law (SCL). Efficiency of implicit subgrid-scale model is tested by three-dimensional Taylor–Green vortex case. According to the test of flow over a cylinder, the influence of the SCL errors has been investigated on curvilinear mesh. Then stall phenomena of thin airfoil NACA64A006 have been simulated by the HILES. The slope of lift curve, the maximum lift and the stall angle are successfully predicted. Moreover, the lift characteristic seems to be satisfactorily captured even after the stall angle. The solutions demonstrate the potential of HILES for simulating complex turbulent flow.

1. Introduction

The need for predictive simulation methods for turbulent flows has led to a significant interest in large-eddy simulations (LES) in recent years. Without LES that captures unsteady behavior of the turbulent flows, accurate result may not be obtained [1]. For example, it is difficult for Reynolds-averaged Navier–Stokes simulations (RANS) model to estimate stall characteristic of thin-airfoil NACA64A006 [1], where the laminar flow separation occurs at the leading edge and the transition causes the turbulent reattachment. The reattachment point gradually moves rearward with increasing angles of attack [2]. The small vortexes shed from the leading edge, which produces strong unsteadiness in the flow. It is difficult for RANS simulations to resolve this feature, and this is the main reason that RANS simulations do not give satisfactory results for this case. According to the limitation of RANS methods, Fujii [1] proposed that LES should be employed for the prediction of thin-airfoil stall characteristics. Furthermore, LES has been successfully applied for the simulations of stall phenomena [3,4].

As well known, high-order scheme has the advantage over low-order scheme for the simulation of turbulent flow containing unsteady vortex shedding. However, the application of high-order compact schemes still has some challenges, such as robustness and grid-quality sensitivity [5,6]. This deficiency can be largely removed by the researches of the Geometric Conservation Law (GCL) [7–11]. The GCL contains surface conservation law (SCL) and volume conservation law (VCL). The VCL has been widely studied for time-dependent grids, while the SCL is merely discussed for finite difference schemes. Recent research [7] shows that the GCL is very important for the application of finite difference schemes on curvilinear grids. If the SCL has not been satisfied, numerical instabilities and even computing collapse may appear on complex curvilinear grids during numerical simulation. In order to fulfill the SCL for high-order finite difference schemes, a conservative metric method (CMM) is derived by Deng et al. [7]. Not long after, a symmetrical conservative metric method (SCMM) [12], which can evidently increase the numerical accuracy on irregular grids, is proposed based on the CMM. According to the principle of satisfying the SCMM, a seventh-order hybrid cell-edge and cell-node dissipative compact scheme (HDCS-E8T7) has been proposed for complex geometry [13]. The HDCS-E8T7 has inherent dissipation to dissipate unresolvable wavenumbers, therefore the filtering is not needed. The properties of HDCS-E8T7 scheme have been systemically analyzed by Deng et. al. [14].

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Based on the HDCS-E8T7, a new high-order implicit large eddy simulation (HILES) is developed following the concept of monotone integrated LES (MILES) [15], i.e. the effects of explicit LES models are imitated by the truncation error of the discretization scheme itself. Although the conception of HILES is similar to that of MILES, HDCS-E8T7 is a new seventh-order compact scheme having inherent dissipation, and the HILES based on HDCS-E8T7 can eliminate the SCL errors, which may contaminate flowfield of the HILES. Moreover, if we use the seventh-order compact scheme for LES without subgrid-scale (SGS) model, i.e., HILES, the accuracy of the flowfield obtained will be seventh-order. However, if subgrid model, for instance, the Smgorinsky subgrid model is added, the accuracy of the LES results will be degenerated to the undesirable second order. If a carefully designed high-order subgrid model is absent, high-order scheme without model may be better than with second-order subgrid models for LES [16].

In this paper, we will investigate the performance of the HILES on curvilinear meshes. Based on the test of flow over a cylinder, the influence of the SCL errors on the HILES has been shown. Then the stall phenomena of thin airfoil NACA64A006 have been simulated by the HILES. In the next section, the governing equations are given. In Section 3, we will give the numerical method comprising three main components: the spatial discretization, the grid metric calculation and the time-integration method. The numerical tests are presented in Section 4.

2. Governing equations

Three dimensional Navier–Stokes equations in computational coordinates may be written as

$$\frac{\partial\widetilde{U}}{\partial t}+\frac{\partial\widetilde{E}}{\partial\xi}+\frac{\partial\widetilde{F}}{\partial\eta}+\frac{\partial\widetilde{G}}{\partial\zeta}=\frac{1}{R_e}\left(\frac{\partial\widetilde{E}_v}{\partial\xi}+\frac{\partial\widetilde{F}_v}{\partial\eta}+\frac{\partial\widetilde{G}_v}{\partial\zeta}\right), \tag{1}$$

where,

$$\widetilde{U}=U/J,$$
$$\widetilde{E}=(\xi_tU+\xi_xE+\xi_yF+\xi_zG)/J,\quad \widetilde{E}_v=(\xi_xE_v+\xi_yF_v+\xi_zG_v)/J,$$
$$\widetilde{F}=\left(\eta_tU+\eta_xE+\eta_yF+\eta_zG\right)/J,\quad \widetilde{F}_v=\left(\eta_xE_v+\eta_yF_v+\eta_zG_v\right)/J,$$
$$\widetilde{G}=(\varsigma_tU+\zeta_xE+\zeta_yF+\zeta_zG)/J,\quad \widetilde{G}_v=(\zeta_xE_v+\zeta_yF_v+\zeta_zG_v)/J,$$

and

$$U=[\rho,\rho u,\rho v,\rho w,\rho e]^T,$$
$$E=\left[\rho u,\rho u^2+p,\rho vu,\rho wu,(\rho e+p)u\right]^T,$$
$$F=\left[\rho v,\rho uv,\rho v^2+p,\rho wv,(\rho e+p)v\right]^T,$$
$$G=\left[\rho w,\rho uw,\rho vw,\rho w^2+p,(\rho e+p)w\right]^T,$$
$$E_v=\left[0,\quad\tau_{xx},\quad\tau_{xy},\quad\tau_{xz},\quad u\tau_{xx}+v\tau_{xy}+w\tau_{xz}+\dot{q}_x\right]^T,$$
$$F_v=\left[0,\quad\tau_{yx},\quad\tau_{yy},\quad\tau_{yz},\quad u\tau_{yx}+v\tau_{yy}+w\tau_{yz}+\dot{q}_y\right]^T,$$
$$G_v=\left[0,\quad\tau_{zx},\quad\tau_{zy},\quad\tau_{zz},\quad u\tau_{zx}+v\tau_{zy}+w\tau_{zz}+\dot{q}_z\right]^T,$$

with viscous stress terms written as

$$\tau_{ij}=\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_i}{\partial x_j}-\frac{2}{3}\delta_{ij}\frac{\partial u_l}{\partial x_l}\right),$$

and heat transfer terms

$$\dot{q}_x=\frac{1}{(\gamma-1)P_rM_\infty^2}\mu\frac{\partial T}{\partial x},$$
$$\dot{q}_y=\frac{1}{(\gamma-1)P_rM_\infty^2}\mu\frac{\partial T}{\partial y},$$
$$\dot{q}_z=\frac{1}{(\gamma-1)P_rM_\infty^2}\mu\frac{\partial T}{\partial z},$$

where γ is the ratio of specific heats, and the viscous coefficient μ can be calculated by the Sutherland's law. The equation of state and the energy function are

$$p=\frac{1}{\gamma M^2}\rho T,e=\frac{p}{(\gamma-1)\rho}+\frac{u^2+v^2+w^2}{2}.$$

In the above u, v and w are the velocity components in x, y and z directions, respectively, p is the pressure, ρ is the density and T is temperature. The non-dimensional variables are defined as $\rho=\rho^*/\rho^*_\infty,(u,v,w)=(u,v,w)^*/V^*_\infty,T=T^*/T^*_\infty,p=p^*/\rho^*_\infty V^{*2}_\infty,\mu=\mu^*/\mu^*_\infty$ respectively, and $M_\infty=u_\infty/\sqrt{\gamma RT^*_\infty},R_e=\rho^*_\infty u^*_\infty r^*/\mu^*_\infty$, $P_r=\mu^*_\infty C_p/\kappa^*_\infty$ are the Mach number, Reynolds number and Prandtl number, r^* is the characteristic length. J is the Jacobi of grid transformation, $\xi_t,\xi_x,\xi_y,\xi_z,\eta_t,\eta_x,\eta_y,\eta_z,\zeta_t,\zeta_x,\zeta_y,\zeta_z$ are grid derivatives. The grid metric derivatives have the conservative form as

$$\begin{cases}\tilde{\xi}_x=\xi_x/J=(y_\eta z)_\zeta-(y_\zeta z)_\eta,\tilde{\xi}_y=\xi_y/J=(z_\eta x)_\zeta-(z_\zeta x)_\eta,\tilde{\xi}_z=\xi_z/J=(x_\eta y)_\zeta-(x_\zeta y)_\eta,\\ \tilde{\eta}_x=\eta_x/J=(y_\zeta z)_\xi-(y_\xi z)_\zeta,\tilde{\eta}_y=\eta_y/J=(z_\zeta x)_\xi-(z_\xi x)_\zeta,\tilde{\eta}_z=\eta_z/J=(x_\zeta y)_\xi-(x_\xi y)_\zeta,\\ \tilde{\zeta}_x=\zeta_x/J=(y_\xi z)_\eta-(y_\eta z)_\xi,\tilde{\zeta}_y=\zeta_y/J=(z_\xi x)_\eta-(z_\eta x)_\xi,\tilde{\zeta}_z=\zeta_z/J=(x_\xi y)_\eta-(x_\eta y)_\xi.\end{cases} \tag{2}$$

3. Numerical method

3.1. Spatial discretization

A seventh-order finite difference scheme HDCS-E8T7 is employed to discretize the equations (1). Considering discretization of the inviscid terms,

$$\frac{\partial\widetilde{U}}{\partial t}+\frac{\partial\widetilde{E}}{\partial\xi}+\frac{\partial\widetilde{F}}{\partial\eta}+\frac{\partial\widetilde{G}}{\partial\zeta}=0, \tag{3}$$

and theirs semi-discrete approximations,

$$\frac{\partial\widetilde{U}}{\partial t}=-\delta_{\rm I}^{\xi}\widetilde{E}-\delta_{\rm I}^{\eta}\widetilde{F}-\delta_{\rm I}^{\zeta}\widetilde{G}. \tag{4}$$

The discretizations $\delta_{\rm I}^{\xi},\delta_{\rm I}^{\eta}$ and $\delta_{\rm I}^{\zeta}$ are the same, thus we only give the discretization in ξ direction. The $\delta_{\rm I}^{\xi}$ of the HDCS-E8T7 is

$$\begin{aligned}\delta_{\rm I}^{\xi}\widetilde{E}_j=&\frac{256}{175h}\left(\widehat{E}_{j+1/2}-\widehat{E}_{j-1/2}\right)-\frac{1}{4h}\left(\widetilde{E}_{j+1}-\widetilde{E}_{j-1}\right)\\&+\frac{1}{100h}\left(\widetilde{E}_{j+2}-\widetilde{E}_{j-2}\right)-\frac{1}{2100h}\left(\widetilde{E}_{j+3}-\widetilde{E}_{j-3}\right),\end{aligned} \tag{5}$$

where, $\widehat{E}_{j\pm1/2}=\widetilde{E}\left(\widehat{U}_{j\pm1/2},\hat{\xi}_{x,j\pm1/2},\hat{\xi}_{y,j\pm1/2},\hat{\xi}_{z,j\pm1/2}\right)$ and $\widetilde{E}_{j+m}=\widetilde{E}\left(U_{j+m},\hat{\xi}_{x,j\pm m},\hat{\xi}_{y,j\pm m},\hat{\xi}_{z,j\pm m}\right)$ are the fluxes at cell edges and at cell nodes, respectively. The numerical fluxes $\widehat{E}_{j\pm1/2}$ may be evaluated by the variables at cell-edges,

$$\widehat{E}_{j\pm1/2}=\widetilde{E}\left(\widehat{U}^L_{j\pm1/2},\widehat{U}^R_{j\pm1/2},\hat{\xi}_{x,j\pm1/2},\hat{\xi}_{y,j\pm1/2},\hat{\xi}_{z,j\pm1/2}\right), \tag{6}$$

where, $\widehat{U}^L_{j\pm1/2},\widehat{U}^R_{j\pm1/2}$ are variables at cell-edge.

$$\begin{aligned}&\frac{5}{14}(1-\alpha)\widehat{U}^L_{j-1/2}+\widehat{U}^L_{j+1/2}+\frac{5}{14}(1+\alpha)\widehat{U}^L_{j+3/2}\\&=\frac{25}{32}(U_{j+1}+U_j)+\frac{5}{64}(U_{j+2}+U_{j-1})-\frac{1}{448}(U_{j+3}+U_{j-2})\\&\quad+\alpha\left[\frac{25}{64}(U_{j+1}-U_j)+\frac{15}{128}(U_{j+2}-U_{j-1})-\frac{5}{896}(U_{j+3}-U_{j-2})\right],\end{aligned} \tag{7}$$

where $\alpha<0$ is the dissipative parameter employed to control the dissipation of the HDCS-E8T7. The corresponding $\widehat{U}^R_{j+1/2}$ can be obtained easily by setting $\alpha>0$. Fig. 1 plots the modified wavenumber of the HDCS-E8T7 with different dissipative parameters. It may be noticed that the resolution of the HDCS-E8T7 is spectral-like.