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Heat and mass transfer inside micro-tubes with uniform injection

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ABSTRACT

This work deals with a numerical study of the transport phenomena along a micro-tube subjected to an injection at the wall. The governing equations expressing the conservation of mass, momentum and energy with first-order slip velocity and temperature jump boundary conditions were solved numerically. The numerical model based on the Finite Volume Method was validated using the available data. The study reveals significant impact of slip velocity and temperature jump conditions on the hydrodynamic and thermal flows. For very small injection rates, all profiles and quantities remain unchanged compared to the impermeable wall case.

1. Introduction

Flows with injection have been widely studied because of their wide range of applications such as in separation processes and biomedical technologies. Uniform injection in micro-ducts was studied by Terrill [1], where heat transfer in laminar flow between parallel porous plates was investigated. It was found that the fully developed Nusselt number decreases with small injection Reynolds number. Pederson and Raithby [2], Kinney [3,4] and others studied the heat transfer for laminar flow inside porous tubes. A large variation of the Nusselt numbers occurs when considering injection. Moussy and Snider [5] investigated the laminar flow over pipes with injection through the porous wall at low Reynolds numbers. Analytical expressions describing the two-dimensional steady-state laminar flow over an array of porous pipes were developed from the solution of the Navier–Stokes equations for the case of low wall Reynolds numbers. Libby et al. [6] investigated the flow development in a tube with injection of light or heavy gas. Recently, flow field and heat transfer at micro-scale have attracted an extensive research interest due to the rapid development of Micro Electro Mechanical Systems (MEMS), biomedical applications, innovative cooling techniques for integrated circuits and separation systems.

Modeling the fluid flow with heat and mass transfer for micro-devices is different from that of the macro-scale systems. The ratio of the mean free path to characteristic length known as Knudsen's number, $Kn = \lambda/L$, defines the region where the continuum assumption is valid and where it becomes no longer valid for the case of gases. For small values of Kn, the fluid behavior can be analyzed using the Navier Stokes equations with no-slip flow boundary conditions. For values of Kn varying between 0.001 and 0.1 the regime is called slip flow regime [7]. However, for Kn higher than 0.1, the continuum description is expected to fail [8]. Other methods using molecular simulations such as the direct simulation Monte Carlo are used for such ranges of Kn numbers.

Many studies investigated the effect of rarefaction and temperature jump condition on the hydrodynamic and thermal fields inside ducts. The majority of these studies have been interested to solve the energy equation when the hydrodynamic flow is considered fully developed for both cases constant wall temperature and constant wall heat flux [7,9]. It is interesting to mention the analytical work of Barron et al. [10] where they extended the original Graetz problem of thermally developing heat transfer in laminar flow through a circular tube to slip flow. Relationships in Knudsen numbers ranging from 0 to 0.12 were developed to describe the effect of slip flow on heat transfer coefficient.

Few studies have had an interest in studying the impact of slip velocity and temperature jump on the hydrodynamic and thermal flows development for permeable walls. In fact, Soundalgekar and Divekar [11] studied laminar slip flow through a uniform circular pipe with small suction, and significant impact of slip velocity is shown on the hydrodynamic flow. Singh and Laurence [12] studied

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Nomenclature

c_p	specific heat at constant pressure (kJ kg^{-1} K^{-1})
D	tube diameter (m)
f_{app}	apparent friction coefficient (–)
h	convective heat transfer coefficient, (W m^{-2} K^{-1})
k	thermal conductivity (W m^{-1} K^{-1})
Kn	Knudsen number (=λ/D) (–)
L	micro-tube length (m)
Nu_x	local Nusselt number (=$h_x D/k$) (–)
P	pressure (Pa)
Pr	Prandtl number, (=$\mu c_p/k$) (–)
r	radial coordinate (m)
Re	Reynolds number, (=$U_{in}\rho R/\mu$) (–)
Re_w	Reynolds wall, (=$V_w \rho D/\mu$) (–)
T	temperature, (K)
U	velocity component in the z-direction (m s^{-1})
V	velocity component in the r-direction, (m s^{-1})
$\widehat{V}$	(=V/V_w) (–)
z	axial coordinate (m)
z^*	dimensionless axial coordinate, (=z/2DRePr) (–)
z^+	dimensionless axial coordinate, (=z/2DRe) (–)

Greek symbols

λ	gas mean free path (m)
μ	dynamic viscosity (Pa s)
γ	ratio of specific heats (–)
ρ	fluid density (kg m^{-3})

Subscripts

in	inlet
w	wall
∞	fully developed

the impact of slip velocity at a membrane surface for ultrafiltration application. Rarefaction has significant impact on concentration polarization.

2. Formulation

A simultaneously developing laminar flow inside a micro-tube with uniform injection is investigated under slip velocity and temperature jump boundary conditions. A number of assumptions are applied: steady state, axi-symmetric and constant fluid properties flow with negligible gravitational forces and negligible viscous dissipation.

The geometry and coordinate system for the duct are shown in Fig. 1.

Considering the following dimensionless quantities:

$$\bar{r} = \frac{r}{R}, \quad \bar{z} = \frac{z}{R}, \quad \bar{V} = \frac{V}{U_{in}}, \quad \bar{U} = \frac{U}{U_{in}}, \quad \bar{P} = \frac{P}{\rho U_{in}^2}, \quad \bar{T} = \frac{T - T_w}{T_{in} - T_w}$$

where R is the radius of the tube, U_{in} and T_{in} are respectively the inlet velocity and the inlet temperature of the fluid, T_w is the wall temperature.

For a flow inside an isothermal micro-tube, the governing equations and the corresponding boundary conditions in non-dimensional form are:

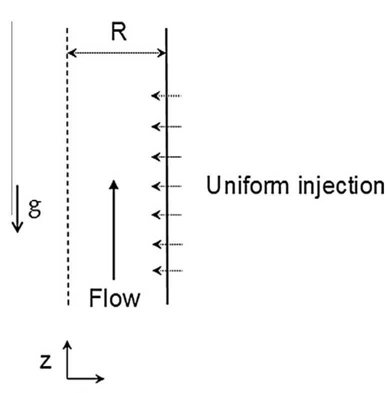

Fig. 1. Geometry and coordinate system of flow domain.

$$\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{V}) + \frac{\partial}{\partial \bar{z}}(\bar{U}) = 0$$

$$\left(\bar{V}\frac{\partial \bar{V}}{\partial \bar{r}} + \bar{U}\frac{\partial \bar{V}}{\partial \bar{z}}\right) = -\frac{\partial \bar{P}}{\partial \bar{r}} + \frac{1}{\mathrm{Re}}\left[\frac{\partial}{\partial \bar{r}}\left(\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{V})\right) + \frac{\partial^2 \bar{V}}{\partial \bar{z}^2}\right]$$

$$\left(\bar{V}\frac{\partial \bar{U}}{\partial \bar{r}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{z}}\right) = -\frac{\partial \bar{P}}{\partial \bar{z}} + \frac{1}{\mathrm{Re}}\left[\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{U}}{\partial \bar{r}}\right) + \frac{\partial^2 \bar{U}}{\partial \bar{z}^2}\right]$$

$$\left(\bar{V}\frac{\partial \bar{T}}{\partial \bar{r}} + \bar{U}\frac{\partial \bar{T}}{\partial \bar{z}}\right) = \frac{1}{\mathrm{RePr}}\left[\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{T}}{\partial \bar{r}}\right) + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right]$$

At the inlet ($z = 0$)

$$\bar{U} = 1; \quad \bar{V} = 0; \quad \bar{T} = 1$$

Symmetry condition at $r = 0$

$$\bar{V} = 0; \quad \frac{\partial \bar{V}}{\partial \bar{r}} = 0; \quad \frac{\partial \bar{T}}{\partial \bar{r}} = 0$$

At $r = R$

$$\bar{V} = \bar{V_w} = V_w/U_{in}$$

$$\bar{U} = -2\beta_v Kn \left.\frac{\partial \bar{U}}{\partial \bar{r}}\right|_{\bar{r}=1}$$

$$\bar{T} = -2\beta_t Kn \left.\frac{\partial \bar{T}}{\partial \bar{r}}\right|_{\bar{r}=1}$$

At the outlet ($z = L$)

$$\frac{\partial \bar{U}}{\partial \bar{z}} = 0; \quad \frac{\partial \bar{V}}{\partial \bar{z}} = 0; \quad \frac{\partial \bar{T}}{\partial \bar{z}} = 0$$

where:

$$\beta_v = (2 - f_v)/f_v; \quad \beta_t = \frac{(2 - f_t)}{f_t}\frac{2\gamma}{1+\gamma}\frac{1}{\mathrm{Pr}}$$

The above coefficients f_v and f_t are known as the tangential momentum accommodation coefficient and thermal accommodation coefficient respectively. These parameters describing the gas–surface interaction are not only functions of the composition, temperature and pressure of the gas but also the gas velocity over the surface, the solid surface temperature and roughness. Their values range from around 0 to 1. For most engineering applications, these coefficients are taken to be close to unity [13,14]. The ratio of β_t to β_v can be introduced as β. Its value can range from 0 to more than 100 [13].