Computers & Fluids 105 (2014) 280–284

Contents lists available at ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

The exterior unsteady viscous flow and heat transfer due to a porous expanding stretching cylinder

Xinhui Si [a,*], Lin Li [a], Liancun Zheng [a], Xinxin Zhang [b], Baiyu Liu [a]

[a] Department of Applied Mathematics, University of Science and Technology, Beijing 100083, China
[b] Department of Mechanical Engineering, University of Science and Technology, Beijing 100083, China

ARTICLE INFO

Article history:
Received 23 March 2012
Received in revised form 3 September 2014
Accepted 15 September 2014
Available online 30 September 2014

Keywords:
Stretching expanding wall
Porous cylinder
Expansion ratio
Dual solutions
Heat transfer

ABSTRACT

This paper presents a numerical solution of the flow and heat transfer outside a stretching expanding porous cylinder. Under this special boundary condition, the governing system of partial differential equations is converted to a set of coupled ordinary differential equations by using suitable similarity transformations, which are solved by a collocation method equivalent to the fourth order mono-implicit-Runge–Kutta method with MATLAB. The main purpose of the present study is to investigate the effects of the different physical parameters, namely the stretching Reynolds number, the permeability Reynolds number,the expansion ratio and the Prandtl number on the velocity and temperature distribution. The results are shown graphically.

1. Introduction

The studies of laminar flow with permeable walls in an expanding or contracting channel or pipe have received considerable attentions in recent years. The earliest work of unsteady flow in a pipe with expanding or contracting wall can be traced back to Uchida and Ohki [1], who showed that the flow equation can be reduced to a single fourth-order nonlinear ordinary differential equation which included expansion ratio. Goto and Uchida [2] analyzed the incompressible laminar flow in a semi-infinite porous pipe whose radius varied with time. Bujurke et al. [3] obtained a series solution for the unsteady flow in a contracting or expanding pipe. Majdalani et al. [4–6], Dauenhauer and Majdalani [7] obtained both numerical and asymptotical solutions for the different permeability Reynolds number. Recently Asghar et al. [8] discussed the flow in a slowly deforming channel with weak permeability using Adomian decomposition method (AMD). Dinarvand et al. [9,10] obtained analytical approximate solutions for the two dimensional viscous flow through expanding or contracting gaps with permeable walls. Boutros et al. [11,12] discussed the same model in a porous channel or pipe with expanding or contracting walls using Lie group method,respectively. Si et al. [13] also obtained analytical solutions for the asymmetric laminar flow in a porous channel with expanding or contracting walls. Using quasilinearization technique, Srinivasacharya et al. [14] solved numerically the flow of a couple stress fluid in a porous channel with expanding or contracting walls. In seeking further generalization, Xu et al. [15] extended the Dauenhauer–Majdalani model to the case in which the wall expansion ratio α is no longer a constant, but rather a time-dependent variable that varies from α_0 to α_1. As a result, they found the time-dependent solutions to approach the steady state very rapidly. Recently, Si et al. [16–19] extended this model to the viscoelastic and micropolar fluid with the same boundary conditions using homotopy analysis method(-HAM). Si et al. [20–22] discussed the existence of multiple solutions for the flow through porous channel or pipe with expanding or contracting walls using a singular perturbation method. Furthermore, the similarity equation [7] describes the unsteady flow of an incompressible fluid in the expanding porous channel. It is presented by White [23] as one of the new exact Navier–Stokes solutions attributed to Dauenhauer and Majdalani.

However, all the above works considered the flow inside expanding or contracting porous channels and tubes. To the best of our knowledge very little reports were found in literatures for the fluids outside the deforming walls. The pioneering work was done by Wang [24], who considered the flow over stretching cylinders and obtained the asymptotic solution for large Reynolds number using perturbation method. Ishak et al. [25] studied the MHD flow and heat transfer due to a stretching cylinder, and they [26] also investigated the effect of uniform suction/injection on flow and heat transfer due to a stretching cylinder. Both problems are

* Corresponding author. Tel.: +86 01062332589.
E-mail address: sixinhui_ustb@126.com (X. Si).

http://dx.doi.org/10.1016/j.compfluid.2014.09.038
0045-7930/

solved numerically by the Keller-box method. Aldos and Ali [27] discussed the MHD free forced convection from a horizontal cylinder with suction and blowing. Recently, Fang et al. [28,29] discussed the unsteady viscous flow outside of an expanding or contracting cylinder and there is no permeability on the boundary.

Motivated by the above-mentioned works, The exterior problem is usually difficult to be solved numerically by a direct solver for Navier–Stokes equations because of its unbounded domain. So it is particularly motivated to use a stream function induced transformation, which turns the governing equations into a boundary value problem of a high order ODE. The problem can then be solved by a BVP solver of ODEs. The effects of different parameters, especially the expansion ratio and the Reynolds number,on the velocity fields and temperature distribution are studied and shown graphically.

2. Formulation of the problem

Consider the laminar flow of an incompressible viscous fluid caused by a porous cylinder, whose radius is $a(t)$ and expands or contracts uniformly at a time-dependent rate $\dot{a}(t)$. As shown in Fig.1, the z-axis is measured along the axis of the cylinder and r-axis is measured in the radial direction. The wall has equal permeability v_w. Assume u and v to be the velocity components in the z and r directions, respectively.We also assume that the stretching velocity of the cylinder wall is proportional to the axial distance from the origin,that is, $u = 2kz$. It is assumed that the surface of the cylinder is at a constant temperature T_w and the ambient fluid temperature is T_∞ , where $T_w > T_\infty$. The viscous dissipation is neglected as it is assumed to be small. Under these assumptions, the governing equations are [24–26]:

$$\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} = \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right), \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2}\right), \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial z} + v\frac{\partial T}{\partial r} = \frac{\kappa}{\rho c}\left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2}\right), \tag{4}$$

where ρ, ν, p, c and κ are density, pressure, kinematic viscosity, specific heat at constant pressure and the coefficient of thermal conductivity, respectively. According to Refs. [11,12,24,29], the boundary conditions are

$$u = 2kz, \quad v = -v_w = -A\dot{a}, \quad T = T_w; \quad r = a(t), \tag{5}$$

$$v \to 0, \quad T \to 0; \quad r \to \infty, \tag{6}$$

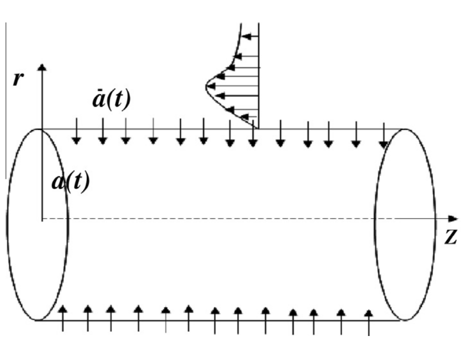

Fig. 1. The porous cylinder with expanding stretching wall.

where $A = \frac{v_w}{\dot{a}}$ is the measure of wall permeability [11,12] and k is a constant representing the stretching strength [29].

Introduce the stream function [12]

$$\psi = \nu z F(\xi, t), \tag{7}$$

where $\xi = \frac{r}{a}$. The velocity and the temperature can be written as

$$u = \frac{\nu z F_\xi(\xi, t)}{a^2 \xi} \quad \text{and} \quad v = -\frac{\nu F(\xi, t)}{a\xi}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}. \tag{8}$$

Substituting ψ, θ into Eqs. (1)–(4), one obtains differential equations. These are

$$\frac{F_{\xi\xi\xi}}{\xi} + \frac{F_{\xi\xi}}{\xi}\left(\frac{F}{\xi} - \frac{1}{\xi} + \alpha\xi\right) - \frac{F_\xi}{\xi}\left(\frac{F_\xi}{\xi} + \frac{F}{\xi^2} - \frac{1}{\xi^2} - \alpha\right) - \frac{a^2}{\nu}\frac{F_{\xi t}}{\xi} = 0, \tag{9}$$

$$\theta_{\xi\xi} + Pr\left(\frac{F}{\xi}\theta_\xi + \theta_\xi \xi\alpha - a^2\nu^{-1}\theta_t\right) = 0, \tag{10}$$

where $Pr = \frac{\rho c \nu}{\kappa}$ is the Prandtl number and $\alpha = \frac{a\dot{a}}{\nu}$ is the wall expansion ratio. Note that the expansion ratio will be positive for expansion and negative for contraction.

A similar solution with respect to both space and time can be developed following the transformation described by Uchida and Aoki [1], Majdalani and Zhou [4] and Dauenhauer and Majdalani [7], respectively. This can be accomplished by considering in the case: α is a constant and $F = F(\xi)$, it leads to $F_{\xi t} = 0$. Here, we assume that $\theta_t = 0$. From a physical standpoint [1,4,7], our idealization is based on a decelerating expansion rate that follows a plausible model,

$$\alpha = \frac{\dot{a}a}{\nu} = \frac{\dot{a}_0 a_0}{\nu} = \text{constant}, \tag{11}$$

where a_0 and $\dot{a}_0$ denote the initial pipe radius and expansion rate, respectively. As a result, the rate of expansion decreases as the internal radius increases. Integrating Eq. (11) with respect to time, the similar solution can be achieved. The result is

$$\frac{a}{a_0} = \frac{v_w(0)}{v_w(t)} = \sqrt{1 + 2\nu\alpha t a_0^{-2}}. \tag{12}$$

Since $v_w = A\dot{a}$ and $A =$ constant [4,7,11,12], then the expression for the injection velocity also can be determined. Under these assumptions, Eqs. (9) and (10) become

$$\frac{F_{\xi\xi\xi}}{\xi} + \frac{F_{\xi\xi}}{\xi}\left(\frac{F}{\xi} - \frac{1}{\xi} + \alpha\xi\right) - \frac{F_\xi}{\xi}\left(\frac{F_\xi}{\xi} + \frac{F}{\xi^2} - \frac{1}{\xi^2} - \alpha\right) = 0, \tag{13}$$

$$\theta_{\xi\xi} + Pr\left(\frac{F}{\xi}\theta_\xi + \theta_\xi\xi\alpha\right) = 0. \tag{14}$$

Introduce a new transformation

$$\eta = \xi^2, \tag{15}$$

then Eqs. (13) and (14) become

$$\eta F''' + F'' + \frac{1}{2}(F''F - F'^2) + \frac{\alpha}{2}(\eta F'' + F') = 0. \tag{16}$$

$$\theta''\eta + \frac{\theta'}{2}(Pr\eta\alpha + PrF + 1) = 0, \tag{17}$$

The boundary conditions are translated into

$$F'(1) = Re^*, \quad F(1) = Re, \quad F'(\infty) = 0, \tag{18}$$

$$\theta(1) = 1, \quad \theta(\infty) = 0, \tag{19}$$

where $Re^* = \frac{ka^2}{\nu}$ is the Reynolds number for the wall stretching and $Re = \frac{a v_w}{\nu}$ is the permeability Reynolds number. Note that Re is positive for injection and negative for suction.

Let

$$f = \frac{F}{Re}, \tag{20}$$