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Nanofluid flow and heat transfer due to a rotating disk

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ABSTRACT

The nanofluid boundary layer flow over a rotating disk is the main concern of the present paper. Unlike the traditional Von Karman problem in which a Newtonian regular fluid is assumed, water-based nanofluids containing nanoparticle volume fraction of Cu, Ag, CuO, Al_2O_3 and TiO_2 are taken into account. The governing equations of motion are reduced to a set of nonlinear differential equations by means of the conventional similarity transformations which are later treated by a spectral Chebyshev collocation numerical integration scheme. The flow and temperature fields as well as the shear stress and heat transfer characteristics are computed for certain values of the nanoparticle volume fraction. A comparative analysis is made in terms of shear stress and cooling properties of considered nanofluids. A mathematical analysis is eventually provided to prove why the nanofluids are advantageous as far as the heat transfer enhancement is concerned. Although the physical features highly rely on the type of the considered nanoparticles, it is found that the heat transfer is greatly enhanced by addition of nanofluid Cu.

1. Introduction

Due to numerous practical applications in aeronautical science as well as in other engineering branches such as thermal-power generating systems, rotating machinery, medical equipments, computer storage devices, gas turbine rotors, air cleaning machines, electronic devices and crystal growth processes, the heat transfer problem over a rotating disk is still being given an extraordinary attention by the researchers. Therefore, the current study also focuses on the heat transfer problem of the boundary layer flow due to a rotating disk, but from a new insight unlike to the classical Von Karman flow with a regular Newtonian fluid, that is, in the presence of five different kinds of nanofluids, which is a hot and emerging topic in the recent literature.

After the pioneering work of Von Kármán [1], the flow due to an infinite rotating disk became a fruitful topic for the aforementioned applications. Among the early investigations are, for instance, [2–4]. A variety of physical features were afterwards explored, which may be referred from the works [5–9] and so on. The very recent articles [10,11] resolve the effects of rotation and radial stretching of the rotating disk, and discusses its potential applications.

Nanofluids, which consist of suspended nanoparticles, can considerably change the transport and thermal properties of the base fluid and thus may enhance thermal conductivity. This is extremely important particularly in microelectro mechanical systems (MEMS) and heat exchangers, since generally a large amount of unwanted heat is generated degrading the performance. To cool such industrial devices, the thermal conductivity of the fluid is increased by adding suspended solid particles, so-called as nanofluids, as coolant. Choi [12] seems to be the first who used the term nanofluids in his research. The publications by Das et al. [13], Wang and Mujumdar [14,15], Kakac and Pramuanjaroenkij [16] and Wong and Leon [17] explain the benefits of the usage of nanofluids in several present practical applications and further discuss the possible usage in futuristic applications. The boundary layer in Newtonian and porous media filled by nanofluids over fixed and moving boundaries have been recently considered by Kuznetsov and Nield [18], Bachok et al. [19,20], Yacob et al. [21], Grosan and Pop [22]. Several nanofluids and their influences on the magnetohydrodynamic slip flow were recently investigated in [23].

Although nanofluids are already involved in some works as cited above, they have not yet been considered over rotating bodies, except the recent work by Pop et al. [24] over a rotating disk, which was unfortunately no error-free as explained in detail later. On the other hand, the rate of heat transfer and hence the effective cooling for some rotating electronic components can be achieved by understanding the proper use of nanofluids. Therefore, the current paper aims at studying the viscous, steady boundary layer flow of some nanofluids due to a rotating disk. Essentially, five types of water-based nanofluids are permitted in the present work: Cu, Ag, CuO, Al_2O_3 and TiO_2. The flow and heat transfer properties influenced by the existence of such nanoparticles are examined thoroughly for the purpose of determining the best performing

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Nomenclature

Greek symbols
η a scaled boundary layer coordinate
θ azimuthal direction in cylindrical polar coordinates
μ dynamic viscosity
ν kinematic viscosity
ρ density
φ nanoparticle volume fraction parameter
τ_r radial shear stress on the wall
τ_θ azimuthal shear stress on the wall
Ω angular speed of the disk

Subscripts and superscripts
w wall conditions
∞ ambient conditions
f for flow
s for solid
nf for nanofluid

nanofluid over a rotating disk in terms of shear stress and heat transfer. A mathematical analysis is further provided to clarify the advantageous of nanofluids over regular one.

The rest of the paper is organized in the following fashion. In Section 2 the governing equations and their similarity reduced forms are given. The results and discussion of the analysis are conducted in Section 3. Section 4 contains the conclusions drawn from the present results. Finally, a transformation is given in Appendix A.

2. Formulation of the problem

The present work is concerned with the traditional flow of rotating disk, first imposed by Von Karman, with the exception that the flow is composed of five different types of nanofluids, which are namely, silver – Ag, copper – Cu, copper oxide – CuO, alumina – Al_2O_3 and titania – TiO_2 together with a base fluid as water. It is assumed that the incompressible, steady and axisymmetric nanofluids flow occur over an impermeable infinite disk that rotates with an angular velocity Ω about its axis z, and cylindrical coordinates (r,θ,z) are adopted. Further, taking into account the nanofluid model proposed by Tiwari and Das [13] (see also Pop et al. [24]), the governing equations of the nanofluid motion and energy can be cast into the form

$$u_r + \frac{u}{r} + w_z = 0, \tag{1}$$

$$uu_r + wu_z - \frac{v^2}{r} = -\frac{1}{\rho_{nf}}p_r + \frac{\mu_{nf}}{\rho_{nf}}\left(u_{rr} + \frac{1}{r}u_r - \frac{u}{r^2} + u_{zz}\right), \tag{2}$$

$$uv_r + wv_z + \frac{uv}{r} = \frac{\mu_{nf}}{\rho_{nf}}\left(v_{rr} + \frac{1}{r}v_r - \frac{v}{r^2} + v_{zz}\right), \tag{3}$$

$$uw_r + ww_z = -\frac{1}{\rho_{nf}}p_z + \frac{\mu_{nf}}{\rho_{nf}}\left(w_{rr} + \frac{1}{r}w_r + w_{zz}\right), \tag{4}$$

$$uT_r + wT_z = \alpha_{nf}\left(T_{rr} + \frac{1}{r}T_r + T_{zz}\right), \tag{5}$$

accompanied by the boundary conditions

$$u = 0, \quad v = \Omega r, \quad w = 0, \quad T = T_w, \quad \text{at} \quad z = 0, \tag{6}$$

$$u \to 0, \quad v \to 0, \quad T \to T_\infty, \quad \text{as} \quad z \to \infty. \tag{7}$$

The velocity field is decomposed into its components (u, v, w) in the directions of increasing (r,θ,z) respectively, the pressure is p, the temperature is T, with T_w and T_∞ denoting wall and ambient conditions. Moreover, the density of the nanofluid is ρ_{nf}, the dynamic viscosity of the nanofluid is μ_{nf} and the thermal diffusivity of the nanofluid is α_{nf}, which are respectively given by

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s,$$
$$(\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s, \quad \frac{k_{nf}}{k_f} = \frac{(k_s+2k_f) - 2\varphi(k_f-k_s)}{(k_s+2k_f) + \varphi(k_f-k_s)}, \tag{8}$$

in which, φ is the nanoparticle volume fraction parameter, the viscosity of the fluid fraction is denoted by μ_f, ρ_f and ρ_s are the densities of the fluid and of the solid fractions, respectively, $(\rho C_p)_{nf}$ stands for the heat capacitance of the nanofluid and the effective thermal conductivity of the nanofluid is given by k_{nf}, approximated by the model given by Oztop and Abu-Nada [25] and is confined to spherical nanoparticles only. The thermophysical properties of the nanofluids are given in Table 1, see for instance [25].

As also emphasized by Pop et al. [24], in nanofluids, there are many theories available to account for the thermophysical properties; the Brownian motion and thermophoresis might also be taken into consideration in a further work. From this aspect, the selection of the thermophysical property itself is a critical task. It should be stated that the thermal conductivity of nanoparticles, particle size, volume fraction and the temperature all have a significant impact on the effective thermal conductivity of the prepared nanofluids. The strong temperature dependence was further modeled by taking the particle size, concentration and temperature into consideration, even changing formulas were suggested for changing nanofluids. However, the Maxwell's theoretical model is adopted in the current work, which is known to predict the thermal conductivity reasonably well for dilute mixtures of relatively large particles in fluids. Hence, in the present work, the selection of thermophysical property model is simply a well-documented and so, a well-studied one for other type of fluid phenomena in the open literature.

By means of the Von Karman's self-similar transformations,

$$\eta = (\Omega/\nu_f)^{1/2}z,$$
$$(u, v, w) = (\Omega r F(\eta), \Omega r G(\eta), (\Omega\nu_f)^{1/2}H(\eta)), \tag{9}$$
$$(p, T) = (2\Omega\mu_f P(\eta), T_\infty + (T_w - T_\infty)\theta),$$

the system (1)–(5) can be reduced to the following ordinary differential equations

$$H' + 2F = 0,$$
$$c_f F'' - F^2 + G^2 - HF' = 0,$$
$$c_f G'' - 2FG - HG' = 0, \tag{10}$$
$$\frac{1}{Pr}c_\theta\theta'' - H\theta' = 0.$$

Table 1
Thermo-physical properties of water and some nanoparticles [25].

	ρ (kg/m^3)	C_p (J/kg K)	k (W/m K)	$\beta \times 10^5$ (K^{-1})
Pure water	997.1	4179	0.613	21
Copper (Cu)	8933	385	401	1.67
Copper oxide (CuO)	6320	531.8	76.5	1.80
Silver (Ag)	10,500	235	429	1.89
Alumina (Al_2O_3)	3970	765	40	0.85
Titanium Oxide (TiO_2)	4250	686.2	8.9538	0.9