Energy Conversion and Management 50 (2009) 1821–1827

Contents lists available at ScienceDirect

Energy Conversion and Management

journal homepage: www.elsevier.com/locate/enconman

Computation of stabilizing PI and PID controllers by using Kronecker summation method

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ARTICLE INFO

Article history:
Received 15 December 2007
Received in revised form 3 August 2008
Accepted 8 March 2009
Available online 11 April 2009

Keywords:
Stabilization
PI control
PID control
Relative stabilization
Kronecker summation
Coefficient uncertainty

ABSTRACT

In this paper, a new method for computation of all stabilizing PI controllers is given. The method is based on the plant model in time domain, and by using the extraordinary feature results from Kronecker sum operation, an explicit equation of control parameters defining the stability boundary in parametric space is obtained. Beyond stabilization, the method is used to shift all poles to a shifted half plane that guarantees a specified settling time of response. The stability regions of PID controllers are given in (k_p, k_i), (k_p, k_d) and (k_i, k_d) plane, respectively. The proposed method is also used to compute all the values of a PI controller stabilizing a control system with uncertain parameters. The proposed method is further extended to determine stability regions of uncertain coefficients of the system. Examples are given to show the benefits of the proposed method.

1. Introduction

Despite continual advances in control theory and development of advanced control strategies, the proportional, integral, and derivative (PID) control algorithm still finds wide applications in industrial process control systems. It has been reported [1] that 98% in of the control loops in the pulp and paper industries are controlled by proportional integral (PI) controllers. Moreover, more than 95% of the controllers used in process control applications are of the PID type [2]. The popularity among industrial practitioners stems from the facts that the PID control structure is simple and its principle is easy to understand and that the PID controllers are deemed to be satisfactory and robust for a vast majority of processes. The primary problem associated with the use of PID controllers is tuning, that is, the determination of PID controller parameters for satisfactory control performance.

Since the primary requirement of the candidate PID controller parameters is that of making the closed-loop system stable, it is often desired to construct the complete sets of stabilizing PID parameters. For instance, as fuzzy PID controllers have been used more and more widely in various control systems [3–5], it is necessary to known the complete sets of stabilizing PID parameters. And as the PID optimization technique develops [6–9], with the complete sets of stabilizing PID controller parameters being available for a given process, it can avoid the time consuming stability check for each set of PID controller parameters in the search process and thereby to save the controller tuning time. Up to now, there has been a great amount of research work on the determination of stabilizing sets of PI (proportional integral) and PID (proportional integral derivative) controllers [10–12]. A complete analytical solution based on the generalized version of the Hermite–Biehler theorem has been proposed [10] for computation of all stabilizing constant gain controllers for a given plant. In Ref. [11], the computation of all stabilizing PI and PID controllers for a given plant by linear programming has been proposed. This approach, besides its numerical efficiency, has also revealed important structural properties of PI and PID controllers. It shows that for a fixed proportional gain, the set of stabilizing integral and derivative gains lie in a convex set. Such an approach can deal with systems that are open loop stable or unstable, minimum or non-minimum phase. However, the computation time for this approach increases in an exponential manner as the order of the system increases, which is a disadvantage of the method. An alternative approach for fast computation of stabilizing PI and PID controllers based on the use of Nyquist plot has been proposed in Refs. [12,13]. Some fast approaches based on the gridding of frequency have been given in Refs. [14,15]. A stability boundary locus approach for the design of PI and PID controllers has been proposed in Ref. [16]. A parameter space approach using the singular frequency concept has been given in Ref. [17] for design of robust PID controllers.

Usually, controller design in classical control engineering is based on a plant with fixed parameters. However, in the real world most practical system models are not known exactly, meaning that

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0196-8904/$ - see front matter © 2009 Published by Elsevier Ltd.
doi:10.1016/j.enconman.2009.03.007

the plant contains uncertainties. Thus, sometimes it is desirable to know the permissible varying range of plant parameters once the controller parameters are fixed. That is, to determine the stability regions of plant parameters. Many approaches to determine the stabilizing sets of PI/PID parameters employ the specific form of PI/PID controllers or the characteristic equation of PI/PID controlled system, thus can not be used to determine the stability regions of plant uncertain parameters.

In this paper, a new method is given for computation of stabilizing PI and PID controllers in the parameter plane. The novel approach makes use of the extraordinary feature of the Kronecker summation operation and we obtain the explicit equation that PI and PID controllers parameters corresponding to the boundary of stability region must satisfy. The proposed method is also used for computation of stabilizing PI and PID controllers for relative stabilization. Actually, the proposed method has a wider application. It is further extended to determine stability regions of uncertain parameters in coefficient space.

The paper is organized as follows: the proposed method is presented in Section 2. In Section 3, the computation of PI controllers for relative stabilization is given. In Section 4, the proposed method is used to determine the stability region of PID controllers. The computation of PI controllers for interval plant stabilization is given in Section 5. In section 6, the proposed method is extended to the determination of stability regions of uncertain coefficients. Concluding remarks are given in Section 7.

2. Stabilization using a PI controller

In early work, many methods proposed to compute the stabilizing sets of PI/PID controllers are based on determining the stability boundary of parameters, the essence of which is to find all the values of parameters which will render pure imaginary roots. Here we study an alternative procedure: Kronecker sum method.

Kronecker summation of two matrices and its properties:

In matrix algebra [18,19] the Kronecker sum of square matrices $M_1(n_1 \times n_1)$ and $M_2(n_2 \times n_2)$ is defined as $M_1 \oplus M_2 = M_1 \otimes I_{n_2} + I_{n_1} \otimes M_2$, where $M_1 \in R^{n_1 \times n_1}, and M_2 \in R^{n_2 \times n_2}$. Here $\oplus$ denotes the *Kronecker summation* and $\oplus$ the *Kronecker product* operations. The most critical feature of the Kronecker summation of M_1 and M_2 is that this new square matrix $M_1 \oplus M_2 \in R^{(n_1 \cdot n_2) \times (n_1 \cdot n_2)}$ has $n_1 \cdot n_2$ eigenvalues which are indeed pair-wise combinatoric summations of the n_1 eigenvalues of M_1 and n_2 eigenvalues of M_2 [18]. That is, the Kronecker sum operation, in fact, induces the "eigenvalue addition" character to the matrices. We take advantage of this feature to obtain the equation that all the values of (k_p, k_i) that render pure imaginary roots must satisfy.

Consider the single input, single output (SISO) control system of Fig. 1 where

$$G(s) = \frac{N(s)}{D(s)} \tag{1}$$

is the plant to be controlled and $C(s)$ is a PI controller of the form

$$C(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s} \tag{2}$$

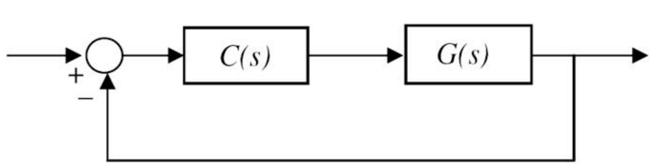

Fig. 1. A SISO control system.

The characteristic equation of the closed-loop system is

$$\begin{aligned} CE(s) &= sD(s) + (k_p s + k_i)N(s) \\ &= f_n(k_p, k_i)s^n + \cdots + f_1(k_p, k_i)s + f_0(k_p, k_i) \end{aligned} \tag{3}$$

Transform Eq. (3) to differential equation matrix and define

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= -\frac{f_0(k_p,k_i)}{f_n(k_p,k_i)}x_1 - \frac{f_1(k_p,k_i)}{f_n(k_p,k_i)}x_2 - \cdots - \frac{f_{n-1}(k_p,k_i)}{f_n(k_p,k_i)} \end{aligned}$$

yields

$$\dot{X} = AX \tag{4}$$

where $\dot{X} = [\dot{x}_1, \ldots, \dot{x}_n]^T,\ X = [x_1, \ldots, x_n]^T$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 0 & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -\frac{f_0(k_p,k_i)}{f_n(k_p,k_i)} & -\frac{f_1(k_p,k_i)}{f_n(k_p,k_i)} & -\frac{f_2(k_p,k_i)}{f_n(k_p,k_i)} & \cdots & \cdots & -\frac{f_{n-1}(k_p,k_i)}{f_n(k_p,k_i)} \end{bmatrix}_{n\times n}$$

The relation between Eqs. (3) and (4) is given by the following equation:

$$CE(s) = f_n(k_p, k_i)\det(sI - A) = 0 \tag{5}$$

It can be seen from Eqs. (3) and (5) that s is the root of Eq. (3) as well as the eigenvalues of matrix A.

Due to the fact that A is a constant matrix, the complex conjugates of s also satisfy Eq. (5).

$$\det(s^* I - A) = 0 \tag{6}$$

Therefore, if $s = j\omega$ is the root of Eq. (3), it must be the eigenvalue of matrix A. And at the same time, $s^* = -j\omega$ is also the root of Eq. (3) and the eigenvalue of matrix A. Since the sum of two eigenvalues $s = j\omega$ and $s^* = -j\omega$ is zero then the Kronecker sum of two matrices must be singular when such $\langle k_p, k_i, \omega \rangle$ correspondence occurs. That is,

$$ACE = \det[A \oplus A] = 0 \tag{7}$$

Note that Eq. (7) does not contain s and is the function of (k_p, k_i). For every value (k_p, k_i) that satisfy Eq. (7), Eq. (3) will have a pair of conjugate imaginary roots or roots at origin. As we know, the imaginary axis and the origin are the only places that the stability shift of the system will occur. Thus, Eq. (7) defines the boundary of the stability region in k_p–k_i plane. The stability boundary may divide the parametric plane into several separate regions. To determine the actual stability regions, choose one point in one separate region, which will result in a polynomial, determine the stability posture of the polynomial by calculating the roots of polynomial. The corresponding system is stable if the polynomial has no right half plane (RHP) root. Then by executing the D-subdivision method [20], the prospective region is a stable one. Repeating the same procedure and test other separated regions and all stability regions can be determined.

2.1. Example 1

Consider the control system of Fig. 1 with transfer function

$$G(s) = \frac{N(s)}{D(s)} = \frac{s^3 + 4s^2 - s + 1}{s^5 + 2s^4 + 32s^3 + 14s^2 - 4s + 50} \tag{8}$$