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Robust adaptive acoustic vector sensor beamforming using automated diagonal loading

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ABSTRACT

In this paper, a novel robust adaptive acoustic vector sensor beamformer based on shrinkage is derived. Unlike many existing methods, the proposed method is completely automatic (or so-called user parameter-free), which means, it do not need the choice of user parameters. The proposed diagonal loading algorithms use shrinkage-based covariance matrix estimates, instead of the conventional sample covariance matrix, in the standard Capon acoustic vector sensor beamforming formulation. The numerical results show that our method is robust against errors on the steering vector and small sample sizes, and meanwhile gives high output signal to interference plus noise ratio (SINR).

1. Introduction

The hydrophone, an omnidirectional underwater microphone, is the most common sensor for listening to underwater sound. Directional acoustic sensors [1,2], however, have many important applications [3,4]. One important class of directional sensors is the acoustic vector sensor which measures the scalar acoustic pressure along with the acoustic particle motion (velocity or acceleration). With this additional vector measurement, these directional sensors feature many advantages over conventional omnidirectional hydrophone sensors [5–19]. A single vector sensor can steer an unambiguous beam in 3D space, albeit typically with course resolution. In any array configuration, they are capable of attenuating spatial ambiguity lobes. In the important special case of a line array configuration, vector sensors can eliminate conical or left/right ambiguity. Vector sensors also provide the ability to "undersample" the acoustic wave without spatial aliasing (sensors spaced greater than half a wavelength apart). Vector sensors feature improved array gain and detection performance over omnidirectional sensors. As a result, vector sensors can be an enabling technology when the size of an array is limited.

Along with their advantages, vector sensors also pose additional practical complexities. Vector sensors are more sensitive than hydrophones to flow noise at low frequencies. This entails careful calibration including scaling the particle motion measurements by the acoustic impedance. Finally, since each acoustic vector sensor has four acoustic channels, adaptive beamforming can become difficult in a sample limited regime, especially with many sensors [20–22].

The Standard Capon Beamformer [sometimes known as MVDR (Minimum Variance Distortionless Response) beamformer] [23–25] is an optimal spatial filter that maximizes the array output signal to interference plus noise ratio, provided that the true covariance matrix and the signal steering vector are accurately known. However, the covariance matrix can be inaccurately estimated due to limited data samples and the knowledge of the steering vector can be imprecise due to look direction errors or imperfect array calibration. Whenever these factors exist, there is a clear performance degradation for the standard Capon beamformer. Therefore, adaptive beamforming approaches robust to small sample sizes problems and steering vector errors are needed [26].

One of the most well-known robust adaptive beamforming approaches is diagonal loading [26]. The main drawback of this method is that there is no clear way to choose the diagonal loading level reliably. Several recent robust adaptive beamformers have been proposed [26–32], which can be regarded as diagonal loading approaches, with the diagonal loading level calculated based on the uncertainty set of the array steering vector. However, we still need to specify the parameter related to the size of the uncertainty set. Indeed, fully parameter-free robust adaptive beamformers are scarce.

We provide alternative approaches for the fully automatic computation of the diagonal loading level. We replace the conventional sample covariance matrix used in the standard Capon acoustic vector sensor beamformer [6,20,22] by an enhanced estimate based on a shrinkage method [33,34], the so-called "shrinkage method" applies a diagonal loading to the covariance matrix where the

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weighting is derived from the data. Numerical examples are presented to compare the performance of the proposed acoustic vector sensor beamformers with that of the standard Capon acoustic vector sensor beamformer in terms of output signal to interference plus noise ratio and signal-of-interest power estimation.

The rest of this paper is organized as follows: Section 2 presents the problem formulation. In Section 3, two proposed shrinkage-based fully automatic robust adaptive acoustic vector sensor beamformers are given. Numerical results are provided in Section 4.

2. Problem formulation

Consider an array comprising M vector sensors and let $\mathbf{R}$ denote the theoretical covariance matrix of the array output vector. We assume that $\mathbf{R} > 0$ (positive definite) has the following form:

$$\mathbf{R} = \sigma_0^2 \mathbf{a}_0 \mathbf{a}_0^* + \mathbf{Q} \tag{1}$$

where σ_0^2 denotes the power of the signal-of-interest, $\mathbf{a}_0$ is the acoustic vector array steering vector [1] of the signal-of-interest with $||\mathbf{a}_0||^2 = 4M$, and $\mathbf{Q}$ is the interference-plus-noise covariance matrix. Under ideal conditions, i.e., $\mathbf{a}_0$ and $\mathbf{R}$ are accurately known, the standard Capon acoustic vector sensor beamformer maximizes the output signal to interference plus noise ratio and the optimal value is $\mathrm{SINR}_{\mathrm{opt}} = \sigma_0^2 \mathbf{a}_0^* \mathbf{Q}^{-1} \mathbf{a}_0$. In practice, the exact covariance matrix $\mathbf{R}$ is unavailable. Therefore, $\mathbf{R}$ is replaced by the sample covariance matrix $\hat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N} y(n) y^*(n)$, with N denoting the number of samples and $y(n)$ representing the nth sample. As N increases, $\hat{\mathbf{R}}$ converges to $\mathbf{R}$, and the value of the corresponding signal to interference plus noise ratio will approach $\mathrm{SINR}_{\mathrm{opt}}$ eventually. However, when $\hat{\mathbf{R}}$ contains samples from signal-of-interest (e.g., in mobile communications applications), the convergence rate of the standard Capon acoustic vector sensor beamformer can be very slow ($N >> 4M$ is required) [6,20]. Consequently, the performance of the standard Capon acoustic vector sensor beamformer degrades substantially in the presence of small sample sizes problems, even when $\mathbf{a}_0$ is exactly known. Moreover, the mismatch between the true and assumed steering vectors ($\mathbf{a}_0$ and $\mathbf{a}$) can also significantly degrade the performance of the standard Capon acoustic vector sensor beamformer.

To improve the performance of the standard Capon acoustic vector sensor beamformer, we replace $\hat{\mathbf{R}}$ by an enhanced covariance matrix estimate based on a shrinkage-based method [33]. The enhanced estimate is obtained by linearly combining $\hat{\mathbf{R}}$ and a shrinkage target (a given matrix with some structure) in an optimal mean-squared error (MSE) sense, which can be done via both analytical and convex optimization approaches as shown in the next section.

3. Shrinkage-based robust adaptive beamforming

A linear shrinkage estimate, which we refer to as the balanced linear combination (BC) [33], has the form:

$$\widetilde{\mathbf{R}} = \alpha \mathbf{I} + (1-\alpha)\hat{\mathbf{R}} \tag{2}$$

where α is the shrinkage intensity, $\widetilde{\mathbf{R}}$ is an enhanced estimate of $\mathbf{R}$ and we use the most commonly employed shrinkage target – the identity matrix $\mathbf{I}$. We also consider a more general linear combination (GLC) [34]:

$$\tilde{\mathbf{R}} = \alpha \mathbf{I} + \beta \hat{\mathbf{R}} \tag{3}$$

The shrinkage parameters for both BC and GLC can be chosen by minimizing (an estimate of) the mean-squared error of the estimator $\widetilde{\mathbf{R}}$, where mean-squared error of $\widetilde{\mathbf{R}}$ is $E\{\|\widetilde{\mathbf{R}} - \mathbf{R}\|^2\}$.

Note that the constraints $\alpha \in [0,1]$ for BC and $\alpha \geq 0$, $\beta \geq 0$ for GLC can be imposed to guarantee that $\widetilde{\mathbf{R}} \geq 0$. Alternatively, we can impose $\widetilde{\mathbf{R}} \geq 0$ directly for both BC and GLC.

We consider the mean-squared error minimization problem for GLC first.

$$\begin{aligned}\mathrm{MSE}(\widetilde{\mathbf{R}}) &= \|\alpha \mathrm{I} - (1-\beta)\mathbf{R}\|^2 + \beta^2\{\|\widehat{\mathbf{R}} - \mathbf{R}\|^2\} \\ &= \alpha^2 M - 2\alpha(1-\beta)\mathrm{tr}(\mathbf{R}) + (1-\beta)^2\|\mathbf{R}\|^2 \\ &\quad + \beta^2 E\{\|\widehat{\mathbf{R}} - \mathbf{R}\|^2\}\end{aligned} \tag{4}$$

where MSE ($\widetilde{\mathbf{R}}$) denotes the mean-squared error of $\widetilde{\mathbf{R}}$.

The optimal values for β and α can be readily obtained by differentiating Eq. (4) with respect to β and α:

$$\beta_0 = \frac{\gamma}{\rho + \gamma} \tag{5}$$

$$\alpha_0 = \nu(1-\beta_0) = \nu \frac{\rho}{\rho+\gamma} \tag{6}$$

where $\rho = E\{\|\widehat{\mathbf{R}} - \mathbf{R}\|^2\}$, $\nu = \frac{tr(\mathbf{R})}{4M}$, and $\gamma = ||\nu\mathbf{I} - \mathbf{R}||^2$. We note that $\beta_0 \in [0,1]$ and $\alpha_0 \geq 0$.

To estimate α_0 and β_0 from the given data, we need an estimate of ρ, which can be calculated as (see [33] for details):

$$\hat{\rho} = \frac{1}{N^2}\sum_{n=1}^{N}\|y(n)\|^4 - \frac{1}{N}\|\widehat{\mathbf{R}}\|^2 \tag{7}$$

Consequently, we can get estimates for α_0 and β_0:

$$\hat{\beta}_0 = \frac{\hat{\gamma}}{\hat{\rho} + \hat{\gamma}} \tag{8}$$

$$\hat{\alpha}_0 = \hat{\nu}(1-\hat{\beta}_0) \tag{9}$$

where $\hat{\nu} = \frac{\mathrm{tr}(\widehat{\mathbf{R}})}{4M}$, and $\hat{\gamma} = \|\hat{\nu}\mathbf{I} - \widehat{\mathbf{R}}\|^2$. Note that $\hat{\alpha}_0$ and $\hat{\beta}_0$ satisfy the constraints $\alpha \geq 0$ and $\beta \geq 0$. In addition, note that $\gamma + \rho = E\{\|\widehat{\mathbf{R}} - \hat{\nu}\mathbf{I}\|^2\}$, an estimate of which is given by $\|\widehat{\mathbf{R}} - \hat{\nu}\mathbf{I}\|^2$. Then we can get alternative estimates of α_0 and β_0 (we need to guarantee that they are nonnegative): we have two methods to obtain the enhanced estimates of the covariance matrix, i.e.

$$\widetilde{\mathbf{R}}_{\mathrm{GLC}} = \hat{\alpha}_0 \mathbf{I} + \hat{\beta}_0 \widehat{\mathbf{R}} \tag{10}$$

and

$$\widetilde{\mathbf{R}}_{\mathrm{BC}} = \hat{\alpha}_0 \mathbf{I} + (1-\hat{\alpha}_0)\widehat{\mathbf{R}} \tag{11}$$

Using one of the above enhanced estimates $\widetilde{\mathbf{R}}$ in lieu of $\hat{\mathbf{R}}$ in the standard Capon acoustic vector sensor beamformer formulation yields the shrinkage-based robust adaptive beamformer: $\widehat{\mathbf{w}} = \frac{\widetilde{\mathbf{R}}^{-1}\mathbf{a}}{\mathbf{a}^*\widetilde{\mathbf{R}}^{-1}\mathbf{a}}$. The resulting beamformer output signal to interference plus noise ratio is given by $\mathrm{SINR} = \frac{\sigma_0^2|\widehat{\mathbf{w}}^*\mathbf{a}_0|^2}{\widehat{\mathbf{w}}^*\mathbf{Q}\widehat{\mathbf{w}}}$, and the signal-of-interest power estimate is $\hat{\sigma}_0^2 = \widehat{\mathbf{w}}^*\widetilde{\mathbf{R}}\widehat{\mathbf{w}}$.

From (10) and (11), we note that the shrinkage-based robust adaptive acoustic vector sensor beamformers are diagonal loading approaches with the diagonal loading levels ($\hat{\alpha}_0/\hat{\beta}_0$ for GLC and $\hat{\alpha}_0/(1-\hat{\alpha}_0)$ for BC) determined automatically from the observed data samples $\{y(n)\}_{n=1}^{N}$.

4. Numerical results

We present below several numerical examples comparing the performance of the shrinkage-based robust adaptive acoustic vector sensor beamformers with that of the standard Capon acoustic vector sensor beamformer. In all examples, we assume a uniform linear array with $M = 8$ acoustic vector sensors and half-wavelength inter-element spacing. The noise is assumed to be spherically isotropic noise [1–9]. A signal-of-interest with a 5 dB power is assumed to impinge on the array from {60°, 90°}, and four interferences, each with a 10 dB power, are assumed to be present at