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Characterization of oscillatory instability in lid driven cavity flows using lattice Boltzmann method

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ABSTRACT

In the present work, lattice Boltzmann method (LBM) is applied for simulating flow in a three-dimensional lid driven cubic and deep cavities. The developed code is first validated by simulating flow in a cubic lid driven cavity at 1000 and 12,000 Reynolds numbers following which we study the effect of cavity depth on the steady-oscillatory transition Reynolds number in cavities with depth aspect ratio equal to 1, 2 and 3. Turbulence modeling is performed through large eddy simulation (LES) using the classical Smagorinsky sub-grid scale model to arrive at an optimum mesh size for all the simulations. The simulation results indicate that the first Hopf bifurcation Reynolds number correlates negatively with the cavity depth which is consistent with the observations from two-dimensional deep cavity flow data available in the literature. Cubic cavity displays a steady flow field up to a Reynolds number of 2100, a delayed anti-symmetry breaking oscillatory field at a Reynolds number of 2300, which further gets restored to a symmetry preserving oscillatory flow field at 2350. Deep cavities on the other hand only attain an anti-symmetry breaking flow field from a steady flow field upon increase of the Reynolds number in the range explored. As the present work involved performing a set of time-dependent calculations for several Reynolds numbers and cavity depths, the parallel performance of the code is evaluated a priori by running the code on up to 4096 cores. The computational time required for these runs shows a close to linear speed up over a wide range of processor counts depending on the problem size, which establishes the feasibility of performing a thorough search process such as the one presently undertaken.

1. Introduction

Lattice Boltzmann method (LBM) is evolving as an alternative method to simulate a variety of fluid flows ranging from low Reynolds number to highly turbulent flows in simple and complex geometries [7,28]. A few advantages of LBM compared to conventional Navier–Stokes simulations are simple numerical implementation, absence of solving a pressure Poisson equation at every time step and ease of parallelization that stems from local collision and streaming operators. These advantages have made it gain popularity over the past few decades and a variety of applications to blood flows [27,17,47], microfluidics [46] and multiphase flows [12,31] can be found in the literature. Fluid flow in a lid driven cavity is a classical benchmark problem that was studied extensively by many researchers owing to its simple geometric configuration and yet showing a variety of flow features such as corner eddies, bifurcation and transition to turbulence. In the following we present a quick review of previous studies on lid driven cavity flows including experiments, Navier–Stokes equations based and lattice Boltzmann method based numerical simulations and thereafter we move onto the goal of the present study.

1.1. Previous studies

Benchmark data on two-dimensional lid driven cavity flow were first reported by Ghia et al. [14] and Schreiber and Keller [33]. These results served as classical benchmark data to perform verification and validation studies of several numerical solvers [41,10,8,36,37]. Very accurate two-dimensional simulations were performed by Botella and Peyret [5] using a spectral method. Later, Hopf bifurcation studies were performed by Goodrich et al. [16], Shen [35], Abouhamza and Pierre [1] and Auteri et al. [3]. These studies have shown that in two-dimensional lid driven cavity flows oscillatory flows can be supported at Reynolds numbers of the order of $O(10^4)$, whereas three-dimensional lid driven cavity flow become unstable at Reynolds numbers an order of magnitude lesser [2]. A review of flow dynamics in the lid driven cavity problem can be found in Shankar and Deshpande [34] and a quick overview of linear stability in lid driven cavity flows can be found in Theofilis [42].

There have been a number of studies on flow in a three-dimensional lid driven cavity [9,15,13,21,32]. Numerical benchmark data

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for a Reynolds number of 1000 in a cubic lid driven cavity were reported by Albensoeder and Kuhlmann [2] using a Chebyshev-collocation in space and Adams–Bashforth backward-Euler scheme in time. Steady state results were reported for a Reynolds number of 865 and 1000 respectively by Tuner et al. [43] and Sun et al. [40] in the code verification and validation studies. Recently, Feldman and Gelfgat [11] numerically predicted the onset of oscillatory instability in a three-dimensional lid driven flow in a cubic cavity and found that the oscillatory instability of the flow sets in via a symmetry-breaking sub-critical Hopf bifurcation approximately at a Reynolds number of 1914. In order to further support these numerical observations, they have also performed experiments using particle image velocimetry (PIV) in Liberzon et al. [24]. Through these experiments they were able to obtain a good agreement for the bifurcation Reynolds number and oscillation frequency.

Coming to numerical studies previously undertaken using lattice Boltzmann method, Patil et al. [30] simulated flow in two-dimensional deep lid driven cavities. They showed that the structure of the primary eddy that gets formed just below the top lid has a drastic change with the Reynolds number, but it is not much affected by the depth of the cavity. They conclude that as the cavity depth increases, the flow-structure near the bottom-wall approaches the limiting case of creeping flow. Recently, Lin et al. [26] have performed simulations of two-dimensional deep lid driven cavities at several aspect ratios using a multi relaxation time lattice Boltzmann method. They noted that at a Reynolds number of 7500 steady state results were obtained for a square cavity whereas unsteady solutions prevailed in the deep cavity flow with rapid changes in the shape and location of the corner vortices. In addition, they captured four primary vortices at a cavity depth of 4 using a multi relaxation time model which was not captured by a single relaxation time model. They conclude that multi relaxation time model is more suited for parallel computations when compared with a single relaxation time model, due to the fact that the former has intense local computations. This is somewhat similar to increasing the problem size per process in a parallel computation so that the communication time does not supersede, thus giving rise to a better parallel speed up curves. In another study, they further analyzed transition in two-dimensional deep lid driven cavity flows using parallelization obtained through Graphical Processing Unit (GPU) [25]. By defining an amplitude coefficient they found that in two-dimensional driven cavity flows first Hopf bifurcation Reynolds number decreases with the increase of the cavity depth.

1.2. Present study

The present effort is motivated following the recent work in identifying the steady-oscillatory transition in three dimensional cubic cavity [11,24] and in two-dimensional deep lid driven cavities [26,25]. The effect of side walls on the transition Reynolds number of flow in a real three dimensional cavity scenario and the suitability of LBM solver to carry out a range of time dependent calculations are the driving factors in undertaking the present work. Therefore, the aim of the present study is to characterize the onset of oscillatory instability in lid driven cavity flow at several cavity depths using a three-dimensional fluid domain. The rest of the paper is organized as follows. Lattice Boltzmann method including the governing equations, types of lattice models and boundary conditions are discussed in Section 2. Validation of the developed solver, selection of the grid size and lattice type, parallel performance of the solver and finally the studies on oscillatory instability on cubic and deep lid driven cavities are presented in Section 3. Finally the results obtained are discussed and conclusions are made in Section 4.

2. Lattice Boltzmann method

2.1. Governing equations

In LBM, the governing equations are the Boltzmann equations given as:

$$\frac{\partial f}{\partial t} + \mathbf{e} \cdot \nabla f(\mathbf{e}, \mathbf{x}, t) = \mathbf{\Omega}(f) \tag{1}$$

where $f(\mathbf{x}, t)$ is particle distribution function that dictates the probability of finding a particle with a velocity $\mathbf{e}$ at a location $\mathbf{x} = (x, y, z)$ at a particular time instant t, and $\mathbf{\Omega}(f)$ denotes the collision term. The single relaxation time Bhatnagar, Gross and Krook (BGK) [4] model is used as the collision operator, as follows:

$$\mathbf{\Omega}(f) = -\frac{1}{\tau}[f(\mathbf{x}, t) - f^{eq}(\mathbf{x}, t)] \tag{2}$$

where τ is the relaxation time taken by the non-equilibrium part of the particles to reach the equilibrium distribution function state represented in the equation as $f^{eq}(\mathbf{x}, t)$. The relaxation time of the particles is related to the microscopic fluid viscosity (ν) as follows:

$$\tau = \frac{1}{2} + 3\nu\frac{\Delta t}{\Delta x^2} \tag{3}$$

where Δx and Δt represent the grid size and time step size after a space–time discretization of the equations. The equilibrium distribution function depends on the local density $\rho(\mathbf{x}, t)$ and the velocity field $\mathbf{u}(\mathbf{x}, t)$. In LBM, the evolution of the particle distribution function governed by the Boltzmann equation is discretized on a lattice type [29]. To derive the lattice Boltzmann equation, the Boltzmann equation needs to be discretized in the microscopic velocity space into a finite number of velocity links $\mathbf{e}_\alpha$ and we have a corresponding discrete set of $\mathbf{f}_\alpha$ and $\mathbf{f}_\alpha^{eq}$. An incompressible counterpart of the BGK model is proposed for low-Reynolds number 2D plane Poiseuille flow [18] and was applied recently using LES in the lattice Boltzmann framework [22,20]. While the standard BGK scheme computes the fluid density ρ and momentum $\rho\mathbf{u}$ as the moments of the distribution function $f(\mathbf{x}, t)$ the incompressible model computes the sum of the distribution function $\delta\rho = \sum_{i=0}^{N} f_\alpha(\mathbf{x}, t)$ that represents small perturbations about a reference density $\rho_0 = O(1)$. The macroscopic density perturbations and the macroscopic velocity are then given by,

$$\delta\rho(\mathbf{x}, t) = \sum_{\alpha=0}^{\alpha_{max}} f_\alpha(\mathbf{x}, t) \tag{4}$$

$$\mathbf{u}(\mathbf{x}, t) = \frac{1}{\rho_0}\sum_{\alpha=0}^{\alpha_{max}} \mathbf{e}_\alpha f_\alpha(\mathbf{x}, t) \tag{5}$$

where α_{max} equals the number of lattice sites depending on the lattice model chosen. For example, α_{max} equals 18 for D3Q19 model and takes a value of 26 for the D3Q27 model. The incompressible BGK scheme reduces significantly the intensity of numerical pressure wave [22] and was used previously for simulating turbulent flows using LBM [20,22]. The equilibrium distribution function $f_\alpha^{eq}(\mathbf{x}, t)$ is defined as a function of the macroscopic quantities $\rho_0, \delta\rho$ and $\mathbf{u}$ as follows:

$$f_\alpha^{eq} = \omega_\alpha\left[\delta\rho + \rho_0\left(\frac{\mathbf{e}_\alpha \cdot \mathbf{u}}{c_s^2} + \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u} \cdot \mathbf{u}}{2c_s^2}\right)\right] \tag{6}$$

Following a standard trapezoidal time–space integration and change of variables [39,19], we finally obtain the fully-discretized lattice Boltzmann equation:

$$f_\alpha(\mathbf{x} + \mathbf{e}_\alpha\Delta t, t + \Delta t) - f_\alpha(\mathbf{x}, t) = -\frac{1}{\tau}\left[f_\alpha(\mathbf{x}, t) - f_\alpha^{eq}(\mathbf{x}, t)\right] \tag{7}$$