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Modeling hypersonic entry with the fully-implicit Navier–Stokes (FIN-S) stabilized finite element flow solver

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ABSTRACT

In this paper, we present a novel scheme for modeling the hypersonic atmospheric entry of large vehicles with an ablative thermal protection system. The Favre-averaged thermochemical nonequilibrium Navier–Stokes equations with Spalart–Allmaras turbulence closure, thermodynamic, chemical kinetic, and quasi-steady ablation model are presented. The numerical method is based on a streamline upwind Petrov–Galerkin (SUPG) stabilized finite element formulation. The formulation and implementation of the finite element approximation are discussed in detail. The performance of the scheme is investigated through a series of increasingly complex applications, culminating in the simulation of a three-dimensional ablating heatshield in transitioning flow.

1. Introduction

Vehicles built for atmospheric entry encounter hazardous thermal environments due to the high velocity of the vehicle. In order to protect the vehicle from such conditions, a thermal protection system (TPS) is employed. One such TPS uses non-reusable, ablating surfaces to reduce and dissipate the large heat fluxes encountered. Because atmospheric entry conditions are notoriously difficult to replicate in the laboratory, mathematical models play a crucial role in TPS development.

To date, several "one-way" coupling approaches have been used wherein the chemically reacting flow field is computed independently of ablating conditions. Then, the computed flow field is used as boundary conditions for a separate ablation model. A number of studies have been conducted using schemes similar to the one-way coupling scheme discussed [1–9]. One of the shortcomings of such an approach is that the chemical effects of the material injected into the flow by the ablator are absent.

More recently, a two-way, loose coupling was developed [10]. In that work, a non-overlapping domain decomposition, operator-splitting approach was used to couple a zero-dimensional surface ablation model to a computed flow field, using a modified version of the DPLR [11] code referred to as DPLR++. The ablation solution provided nonlinear Dirichlet boundary conditions for each cell on the surface. A flow time step with these conditions updated the flow domain results, which would then provide inputs to the next ablation solve. This iteration would proceed to steady state.

Although this approach obtains a coupled reacting flow solution including chemistry due to injected ablation products, it suffered from slow convergence to steady state. This is a serious problem when one wishes to find steady state solutions for many values of model parameters in an uncertainty quantification analysis. The convergence of the operator splitting approach would become particularly relevant when combined with fully implicit flow formulations such as the one in [12] in which the operator splitting convergence would become the primary limiting factor to time step size and thus to performance.

Closer to the present work is that of Maclean and coworkers. In [13], they present a surface ablation model, identical to that used in earlier work in [14,10], which is implicitly coupled to the finite volume code DPLR. They show two-dimensional results, particularly focused on arc jet applications, but no three-dimensional results.

In this work, we discuss a fully implicit solution of the compressible reacting Navier–Stokes equations in thermal nonequilibrium with surface ablation. The ablation model is identical to that used in [10] and discussed in [14]. Section 2 describes the mathematical model used. Then, Section 3 highlights the stabilized finite element method used throughout this work. In particular, we draw attention to alternative forms of the stabilization parameter τ_{SUPG}

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that can have a dramatic impact on the robustness of the solution algorithm. In Section 4.1, we describe the implementation and verification of the finite element formulation in the code `FIN-S`, built on the `libMesh` finite element library [15], and the salient features enabling more effective solution convergence. Section 4.3 details the implicit algorithm. A variety of two-dimensional and three-dimensional results are in Section 5, including an axisymmetric capsule with an ablating surface. In particular, we demonstrate substantial performance and convergence improvements over the previous operator-splitting scheme [10]. Finally, we conclude in Section 6.

2. Mathematical model

In this section, we describe the details of the mathematical model. We start with the conservation equations, then discuss the turbulence model used. Next, we detail the thermochemistry and transport models considered in this work. Finally, we rewrite the equations in system form, which are more natural for considering weak formulations.

2.1. Conservation equations

The conservation laws for this class of problems are the compressible, reacting, Navier–Stokes equations in thermal nonequilibrium (see, e.g., [16,17]):

$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \boldsymbol{u}) = \nabla \cdot (\rho \mathcal{D}_s \nabla c_s) + \dot{\omega}_s \tag{1}$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) = -\nabla P + \nabla \cdot \boldsymbol{\tau} \tag{2}$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} H) = -\nabla \cdot \boldsymbol{q} + \nabla \cdot \left(\rho \sum_{s=1}^{N_s} h_s \mathcal{D}_s \nabla c_s \right) + \nabla \cdot (\boldsymbol{\tau}\boldsymbol{u}) \tag{3}$$

$$\frac{\partial \rho e_V}{\partial t} + \nabla \cdot (\rho e_V \boldsymbol{u}) = -\nabla \cdot \boldsymbol{q}_V + \nabla \cdot \left(\rho \sum_{s=1}^{N_s} e_{Vs} \mathcal{D}_s \nabla c_s \right) + \dot{\omega}_V \tag{4}$$

Here, ρ_s is the density of species s, t is time, $\boldsymbol{u}$ is the velocity vector, ρ is the density of the chemical mixture, $\mathcal{D}_s$ is the diffusivity of species s, c_s is the species mass fraction, $\dot{\omega}_s$ is the net mass production of species s due to chemical reactions, P is the pressure, $\boldsymbol{\tau}$ is the viscous stress tensor, E is the specific total energy, $H = h + \frac{1}{2}\boldsymbol{u}\cdot\boldsymbol{u}$ is the total enthalpy, h is the mixture specific enthalpy, specific enthalpy, $\boldsymbol{q}$ is the heat flux vector, h_s is the species specific enthalpy, e_V is the specific vibrational energy, $\boldsymbol{q_V}$ is the vibrational heat flux vector, e_{Vs} is the species specific vibrational energy, and $\dot{\omega}_V$ is the vibrational energy production due to the creation of molecules with some vibrational/electronic energy as well as the transfer of energy between the various modes in the gas. The special case of thermal equilibrium is recovered simply by omitting (4).

Note that we have assumed species diffusion velocities obey Fick's Law:

$$\rho_s \boldsymbol{u}_s = -\rho \mathcal{D}_s \nabla c_s \tag{5}$$

Furthermore, we assume the gas behaves as a Newtonian fluid obeying Stokes' hypothesis:

$$\boldsymbol{\tau} = \mu\left(\nabla \boldsymbol{u} + \nabla^T \boldsymbol{u}\right) - \frac{2}{3}\mu(\nabla \cdot \boldsymbol{u})\boldsymbol{I} \tag{6}$$

where μ is the fluid viscosity. Finally, we currently neglect radiative heat transfer and assume Fourier's law for heat conduction:

$$\boldsymbol{q} = -k\nabla T - k_V \nabla T_V \tag{7}$$

$$\boldsymbol{q}_V = -k_V \nabla T_V \tag{8}$$

where T, T_V are, respectively, the translational and vibrational temperatures while k, k_V are the translational and vibrational thermal conductivity.

2.2. Turbulence modeling

Although (1)–(4), with additional thermochemistry and transport models, comprise a complete set of equations for the modeling of hypersonic flows, the computational resources necessary to adequately resolve all the scales of turbulence are not readily available. Instead, the governing Eqs. (1)–(4) are averaged to produce the Favre-averaged Navier–Stokes (FANS) equations. These mean equations are unclosed due to averaging over nonlinear terms, which necessitates the addition of a turbulence model to close the system. In this work, the eddy viscosity concept is used to represent the effects of turbulence. In particular, the model equations take the same form as (1)–(4) but the variables are mean quantities and the transport properties are augmented by "eddy" transport variables as follows:

$$\mu \to \mu + \mu_t,$$
$$\mathcal{D}_s \to \mathcal{D}_s + \mathcal{D}_t,$$
$$k \to k + k_t,$$
$$k_V \to k_V + k_{V,t}.$$

The eddy mass diffusivity $\mathcal{D}_t$ and thermal conductivities k_t and $k_{V,t}$ are computed from the eddy viscosity μ_t:

$$\mathcal{D}_t = \frac{\mu_t}{\rho Sc_t},$$
$$k_t = \frac{\mu_t C_p}{Pr_t},$$
$$k_{V,t} = \eta \mu_t C_v^{vib},$$

where Sc_t, Pr_t, and η are model parameters and the specific heats are discussed in Section 2.4.1. To complete the model, the eddy viscosity μ_t must be defined. Here, the Spalart–Allmaras (SA) one-equation model [18] is used to compute the eddy viscosity. This model is written in terms of a single PDE for the working variable ν_{sa}, which is algebraically related to the eddy viscosity. Specifically,

$$\frac{\partial \rho \nu_{\text{sa}}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \nu_{\text{sa}}) = c_{b1} S_{\text{sa}} \rho \nu_{\text{sa}} - c_{w1} f_w \rho \left(\frac{\nu_{\text{sa}}}{d}\right)^2 + \frac{1}{\sigma}\nabla \cdot [(\mu + \rho \nu_{\text{sa}})\nabla \nu_{\text{sa}}] + \frac{c_{b2}}{\sigma}\rho \nabla \nu_{\text{sa}} \cdot \nabla \nu_{\text{sa}}. \tag{9}$$

Further, the turbulence model has the following algebraic closure relationships:

$$\mu_t = \rho \nu_{\text{sa}} f_{v1} \tag{10}$$

$$f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3} \tag{11}$$

$$f_{v2} = 1 - \frac{\chi}{1 + \chi f_{v1}} \tag{12}$$

$$\chi = \frac{\nu_{\text{sa}}}{\nu}, \tag{13}$$

$$f_w = g\left(\frac{1 + c_{w3}^6}{g^6 + c_{w3}^6}\right)^{1/6} \tag{14}$$

$$g = r + c_{w2}\left(r^6 - r\right) \tag{15}$$

$$r = \frac{\nu_{\text{sa}}}{S_{\text{sa}} \kappa^2 d^2} \tag{16}$$

The remaining closure function, S_{sa}, is slightly different from that specified in the original model. In the original formulation [18], S_{sa} is given by

$$S_{\text{sa}} = \Omega + \frac{\nu_{\text{sa}}}{\kappa^2 d^2} f_{v2}, \tag{17}$$