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An in-depth study on vortex-induced vibration of a circular cylinder with shear flow

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ABSTRACT

To investigate the in-depth mechanism of vortex-induced vibration of a circular cylinder with shear flow, in this paper, with the use of exponential-polar coordinate attached on the moving cylinder, the stream function-vorticity equations of vortex-induced vibration, the initial/boundary conditions and distribution of hydrodynamic force together with cylinder motion equation in shear flow are deduced, the hydrodynamic force consists of inertial force, the vortex-induced force and viscous damping force. Similarly, the cylinder motion equation with virtual mass is induced where the virtual mass consists of the cylinder mass, the potential added mass and the apparent added mass induced by viscosity. Our numerical results revealed that there are three factors affecting fluid-structure interactions from the fixed cylinder to its steady vibration: The first is the vortex shedding where one side shear layer of cylinder strengthens with the effect of the dominated vortex. The second is the vibration of cylinder which pushes the fluid on the pressure side and pumps that on the suction side. The third is the vortexes strengthen in one side and weaken in the other side together with the shift of front stagnation point with the effect of background vortex which is generated by shear flow. The character of vortex-induced vibration in shear flow are affected by the above three factors.

1. Introduction

Fluid–structure interactions [1–9] occur in many engineering fields. These interactions give rise to complicated vibrations of the structures and can cause structural damage under certain unfavorable conditions. For a cylinder mounted on flexible supports, the fluctuating forces induced by changing vortex shedding cause the cylinder to vibrate. Next, the vibrating cylinder alters the flow field, which, in turn changes the flow-induced force. The vibration of the cylinder can increase further until a limiting behavior is reached. This vortex-induced vibration (VIV) phenomenon is one of the most basic and revealing problems.

Representative experimental studies on VIV include those of Feng [10], Griffin and Koopmann [11], Griffin [12], Griffin and Ramberg [13], Brika and Laneville [14] and Hover et al. [15], in which classic lock-in was observed, whereas the shedding frequency coincided with the natural structure frequency. The cylinder experiences significant vibration only with lock-in, and the vibration amplitude has a strong relationship with the phase difference between the lift force and the cylinder motion. However, recently, the experimental results of Gharib [16] and Khalak and Williamson [17] provided examples of significant flow-induced vibration without lock-in and suggested that the lock-in of VIV is dependent on the values of the cylinder/fluid mass ratio. Recently, Franzini et al. [18], Lam and Zou [19] and Korkischko and Meneghini [20] focus on the interaction of multiple cylinders. It was found that the gap or arrangement has a significant effect on the response of the VIV system. Moreover, the experimental results of flow around a circular cylinder with moving surface boundary layer control (MSBC) have been presented which has the advantages of drag reduction and vibration suppression [21].

Various numerical approaches have also been proposed to treat the fully coupled problem involved in VIV. It can be divided into two broad categories usually. For the one, Navier–Stokes equations are solved directly, such as the direct numerical simulation [22–25], the spectral element spatial method [26,27] and the finite element method [28–30]. For the other, the flow field is obtained by solving the vorticity transport equations, where the usual assumption is a two-dimensional laminar flow, such as VIC (vortex-in-cell) method [31] and the viscous-vortex method [28,32]. It has been shown from these studies that in a great majority of the cases, the response is essentially sinusoidal. The lock-in phenomenon was discussed, and the vortex-induced vibrations on a circular cylinder and the associated phenomena, such as the response of the

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cylinder, the unsteady lift and drag on the cylinder, the vortex shedding frequency, and the effects of the cylinder motion on the vortex structure in the wake [33,34], were examined further. Recently, numerical simulation for the interaction of multiple cylinders was also performed [19,31,35–38]. The flow interference between two circular cylinders, one stationary and the other free to oscillate in the transverse direction were studied [39]. Moreover, two-degrees-of-freedom vortex-induced vibrations of a circular cylinder close to a plane boundary were investigated [40,41].

The progress made during the past two decades on VIV has been reviewed [42,43]. It is clear that the investigation of fluid–structure interactions as a fully coupled problem are far from complete, and there still remain some uncertainties, such as added mass, force decomposition and their effects on the characteristics of the fluid–structure system. Therefore more investigations on an in depth analysis are necessary.

In this paper, the VIV with shear flow is investigated numerically. The problems discussed are described by the stream function–vorticity equations in coordinates attached on the moving cylinder, coupled with the cylinder motion equation. The hydrodynamic forces on the cylinder surface are completely derived mathematically from the governing equations, which consists of vortex-induced force, inertial force and viscous damping force. It is worth noting that the viscous damping force related directly with Reynolds number, giving the viscous contribution on damping cylinder vibration, is usually neglected in some earlier publications due to the widely accepted Lighthill's force decomposition. In addition, added mass, one of the most confused parameters in mathematical expression, has also been derived mathematically by the assumption of the sinusoidal response, which consists of the cylinder mass, the potential added mass and the apparent added mass induced by viscosity. Finally the phase angles among the cylinder displacement, the total lift force and the vortex-induced lift force are formulated related with the amplitudes of the vibrating cylinder and force.

The equation of vorticity transport is solved by using the Alternative-Direction Implicit (ADI) algorithm. And the equation of stream function is integrated by means of a Fast Fourier Transform (FFT) algorithm [44–47]. Meanwhile the cylinder motion is calculated by the Runge–Kutta method [44].

In order to reveal the deeper understanding of the fluid–structure interaction of VIV after the limiting behavior has been reached, the effects of the instantaneous wake geometries and the corresponding cylinder motion on the hydrodynamic forces distributed on the cylinder surface are discussed numerically in one entire periodic of vortex-shedding. The drag–lift phase diagram is employed to discuss the effects of VIV on drag and lift forces, which not only denotes the corresponding fluctuation of the drag and lift over a complete time period, but also implies the detail information of the flow patterns. The variations of some characteristics of the fluid–structure interaction of VIV, such as displacement and amplitude of VIV cylinder, in-phase and out-of-phase components of the fluid force, phase angle between the lift force and the cylinder displacement, added mass, lift and drag, etc., in this evolution process are also described.

2. Governing equations

A circular cylinder placed in a flow experiences alternating lift and drag due to vortex-shedding. Thus, a cylinder mounted on flexible supports is expected to undergo vibrations, known as vortex-induced vibration (VIV). The vibration of cylinder affects the flow around the cylinder which, in turn, changes the induced hydrodynamic forces on the cylinder and hence the structure response. This is a fully coupled fluid–structure interaction problem.

The stream–vorticity equations in the exponential–polar coordinates system (ξ, η), $r = e^{2\pi\xi}$, $\theta = 2\pi\eta$, attached on the moving cylinder, for an two-dimensional incompressible fluid become

$$H\frac{\partial \Omega}{\partial t} + \frac{\partial (U_r\Omega)}{\partial \xi} + \frac{\partial (U_\theta \Omega)}{\partial \eta} = \frac{2}{\mathrm{Re}}\left(\frac{\partial^2 \Omega}{\partial \xi^2} + \frac{\partial^2 \Omega}{\partial \eta^2}\right) \quad (1)$$

$$\frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \eta^2} = -H\Omega \quad (2)$$

where the stream function ψ is defined as $\frac{\partial \psi}{\partial \eta} = U_r = H^{\frac{1}{2}} u_r, -\frac{\partial \psi}{\partial \xi} = U_\theta = H^{\frac{1}{2}} u_\theta$, while the vorticity Ω is defined as $\Omega = \frac{1}{H}\left(\frac{\partial U_\theta}{\partial \xi} - \frac{\partial U_r}{\partial \eta}\right)$, with u_r and u_θ the velocity components in r and θ directions, respectively. Furthermore, $H = 4\pi^2 e^{4\pi\xi}$, $\mathrm{Re} = \frac{2u_\infty a}{v}$, u_∞ is the free-stream velocity, v is the kinematic viscosity, a is the cylinder radius, the non-dimensional time is $t = \frac{t^* u_\infty}{a}$. It is noteworthy that the above equations (Eqs. (1), (2)) have the same form as that in the absolute coordinate system.

The sketch of shear flow with a linear velocity profile $u = u_\infty + Gy$ [48] over a cylinder in two-dimensional approach is shown in Fig. 1, where u_∞ is the free-stream velocity at the center-line $\theta = 0$, y is the coordinate in the lateral direction with $y = 0$ at the center of the cylinder, and G is the lateral velocity gradient.

The shear rate K is defined as $K = 2Ga/u_\infty$, a is the cylinder radius. Only the case of a positive shear rate ($K > 0$) is discussed in the paper, which implies that the flow velocity on the upper side is faster than that on the lower side.

An analytic solution [49] for shear flow shown in Fig. 1 can be derived when the flow field is considered to be inviscid initially, that is

$$\psi = -2sh(2\pi\xi)\left[\sin(2\pi\eta) + \frac{K}{2}\left(2ch(2\pi\xi)\cos(4\pi\eta) - e^{2\pi\xi}\right)\right]$$
$$\text{and} \quad \Omega = K \quad (3)$$

At $t > 0$, under the action of the vortices, the constrained cylinder begins to vibrate in the transverse direction. In terms of the Galilean velocity decomposition and the stream function definition, we have

$$\psi = \psi' + \frac{dl(t)}{dt} e^{2\pi\xi}\cos(2\pi\eta)$$

where superscript "′" represents the absolute coordinate, no superscript denotes the coordinate fixed on the cylinder moving with the velocity $\frac{dl(t)}{dt}$, l is the dimensionless cylinder displacement in the transverse direction.

Defining the relative angle of the incoming flow direction $\theta_0 = \tan^{-1}\left[\frac{dl(t)}{dt}\right]$, then

$$\psi = \psi' + (\tan\theta_0)e^{2\pi\xi}\cos(2\pi\eta) \quad (4)$$

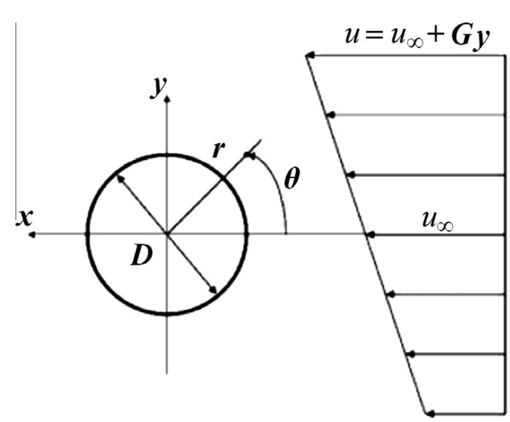

Fig. 1. Sketch of shear flow over circular cylinder.