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Modified SLAU2 scheme with enhanced shock stability

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ABSTRACT

Two alternate modifications are proposed to the dissipation term in the mass flux computation of the low dissipation AUSM scheme (SLAU2) developed recently by Kitamura and Shima [1,2]. These modifications are required to remove the odd–even type instability that results in lateral oscillations behind oblique shocks predicted by MUSCL based higher order versions of SLAU2. The first modification involves switching between the original term in SLAU2 and one similar to corresponding term in AUSM^{+}-up. The second modification involves use of density gradient aligned velocity instead of total velocity in SLAU2 (or face normal velocity as in AUSM^{+}-up) in calculation of Mach number that is required for computing this term. It is observed that the second alternative not only delivers better results but also has a more easily differentiable numerical flux that enables easier implicit computations while not altering the simplicity of original SLAU2. The method also renders SLAU2 with a good balance between shock stability and contact capturing ability.

1. Introduction

The Advection Upstream Splitting Method (AUSM) based on cell interface advection Mach number is considered to provide, simultaneously, the accuracy of flux difference splitting methods and the robustness of flux vector splitting methods. It was first proposed by Liou and Steffen [3] and modified several times [4–6] to address the various pathological problems associated with high speed flow solvers. A comprehensive review of AUSM related work was done by Liou [7] and there have also been some more developments since then [6,8]. To minimize numerical dissipation, Shima and Kitamura developed a new AUSM version called SLAU (Simple Low dissipation AUSM) [1,9]. SLAU2 [2] was developed later to deal with the high speed flows and the shock capturing problem. While SLAU2 has many advantages and it is quiet stable, its MUSCL based second order extension predicted saw-tooth type oscillations in density field behind oblique shock for compression ramp. It appears that this susceptibility becomes evident only when numerical shock thickness is low as it is the case in the second order version. It is also possible that the oscillations result from the multidimensional implementation or higher order extension procedure rather than the scheme itself. In a comparative study of many high resolution schemes, Liska and Wendroff [10] showed that while Piecewise Parabolic Method (PPM) [11] is one of the best schemes to capture the fronts in the Woodward–Collela one-dimensional interacting blast waves test problem, it develops unphysical wiggles while a simulating circular blast wave (test case suggested by Toro). Most contact line resolving flux split schemes like HLLC [12] suffer from the so called odd–even instability. EFMO (equilibrium flux method with Osher intermediate states) scheme [13,14] has been shown to be robust even at Mach number of 100 for flow around a cylinder [13], it suffers from odd–even instability [15] in the Quirk test [16]. Shima and Kitamura [17] showed that SLAU with the van Albada limiter predicted post shock oscillations for this same problem which were attributed to pressure difference related damping term becoming zero in supersonic flows. A Shock Detecting SLAU (SD-SLAU) scheme was proposed in which SLAU is replaced by LSHUS (low dissipation simple high resolution upwind scheme) at the shock front as a fix. Although, multidimensional limiting procedures are available [18] to overcome problems associated with increasing spatial order of accuracy, the problem of oscillations in SLAU2 with higher order accuracy is due to the scheme itself rather than the MUSCL procedure. In fact, the same MUSCL procedure was adopted on other schemes to obtain oscillation free solutions.

Several plausible explanations were hypothesized and tested as to why the oscillations appeared when using SLAU2 scheme in the present study. In addition to trying out all known second order TVD limiters, different combination of primitive variables were considered for interpolation in the MUSCL procedure. Reconstruction procedure using interpolation of conserved variables, change in mesh skewness at the corner and a problem with wall boundary conditions which could propagate along the length of the shock

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were considered. As a simple solution, a damping term that is a weighted average of the original term in SLAU2 and like one in AUSM+-up was attempted. This approach is more seamless than abrupt switching and was deemed more suited for unsteady simulations with moving shocks. Also, the weights are based on a parameter that characterizes unphysical oscillations rather than a shock sensor. While this procedure was promising in most of the test problems considered, it compromised the normal shock related robustness of the SLAU2 in the Quirk's test [16]. So, an alternative procedure which uses density gradient aligned velocity to calculate Mach number involved in pressure damping term was constructed and tested.

In the following sections, the SLAU2 scheme and the MUSCL scheme are explained. The two proposed modifications to prevent oscillations behind shocks are presented next. The latter modification is shown to suppress unphysical oscillations without losing the established robustness of the SLAU2 scheme. This is demonstrated through simulations of some standard test problems for Euler equations.

2. Numerical method

An explicit second order Runge–Kutta scheme (see Appendix A) was used for temporal integration of the governing equations. For spatial discretization, the SLAU2 scheme from an earlier study [2] was chosen. MUSCL procedure with minmod limiter is used for extension to higher order accuracy in space because it is most dissipative and thus less likely to amplify oscillations. The multidimensional limiting process [18] is a linear multiple of the minmod limiter and, therefore, this limiter is more likely to retain multidimensional monotonicity better than any other.

2.1. SLAU2 scheme

The properties on left and right sides of the face are denoted using subscripts "L" and "R" respectively. The pressure on the face used for computing pressure flux is obtained using the following equations.

$$p_{face} = \frac{p_L + p_R}{2} + \frac{f^+(M_L) - f^-(M_R)}{2}(p_L - p_R) + \frac{\rho_L + \rho_R}{2} c_{1/2}[f^+(M_L) + f^-(M_R) - 1] * \sqrt{\frac{K_L + K_R}{2}} \quad (1)$$

$$f^{\pm}(M) = \frac{(M \pm |M|)}{2M}, \quad if\ |M| \geqslant 1 \\ = \frac{1}{4}(2 \mp M)(M \pm 1)^2, \quad otherwise \quad (2)$$

In above equations, ρ, p and K denote the density, pressure and specific kinetic energy respectively. M represents face normal Mach number computed using velocity normal to the face and $c_{1/2}$ is interfacial speed of sound. Kitamura and Shima [2] noted that the SLAU2 is not very sensitive to the specification of the interfacial of sound. A simple geometric mean of the values on either sides is used after verifying the fact that replacing it with more complex calculation using critical speed of sound (as in AUSM+-up) has negligible impact on the results. The mass flux across the face is computed using following equations

$$\widehat{M} = \min\left[1, \frac{1}{c_{1/2}}\sqrt{\frac{K_L + K_R}{2}}\right] \quad (3)$$

$$\chi = (1 - \widehat{M})^2 \quad (4)$$

$$g = \max(\min(M_L, 0), -1)\min(\max(M_R, 0), 1) \quad (5)$$

$$|V_n|^+ = (1 - g)|V_n| + g|c_{1/2}M_L| \quad (6)$$

$$|V_n|^- = (1 - g)|V_n| + g|c_{1/2}M_R| \quad (7)$$

$$V_n = c_{1/2}\frac{\rho_L|M_L| + \rho_R|M_R|}{\rho_L + \rho_R} \quad (8)$$

$$\dot{m} = \frac{1}{2}\left[\rho_L(M_L c_{1/2} + |V_n|^+) + \rho_R(M_R c_{1/2} - |V_n|^-)\right] - \frac{\chi}{2}\frac{\Delta p}{c_{1/2}} \quad (9)$$

Δp represents the jump in pressure across the cell face. Velocity vector and total specific enthalpy from upstream side along with mass flux from above equations are used to compute the convective fluxes.

2.2. MUSCL procedure

To extend the order of accuracy, dependent variable values just to the left and right of the face are computed using higher order interpolations. Primitive variables are extrapolated from cell centers to cells faces. Specifically, velocity components, density and temperature are chosen. The results remained almost unchanged when temperature was replaced by pressure. The extrapolation procedure for the face $(i+1/2, j, k)$ which separates cells (i, j, k) and $(i+1, j, k)$ on an uniform computational mesh is as follows (indices "j" and "k" are dropped for the sake of clarity).

$$U_L(i + 1/2) = U_i + \frac{\phi(r_L)}{2}[U_{i+1} - U_i] \quad (10)$$

$$U_R(i + 1/2) = U_{i+1} - \frac{\phi(r_R)}{2}[U_{i+1} - U_i] \quad (11)$$

U in above equations represents a primitive variable. r_L and r_R determine the monotonicity of the variables on either sides of the face and are determined using following equations.

$$r_L = \frac{U_i - U_{i-1}}{U_{i+1} - U_i} \quad (12)$$

$$r_R = \frac{U_{i+2} - U_{i+1}}{U_{i+1} - U_i} \quad (13)$$

Higher order computations of face values lead to non-monotonic behavior around sharp fronts, so a limiter function, φ is used to lower the order locally. Negative r indicates non-monotone behavior and the limiter function is set to zero preventing higher order extrapolation. A min-mod limiter which is second-order TVD and also ensures multi-dimensional monotonicity [18] more than any other limiter is used here.

$$\phi(r) = \max(0, \min(1, r)) \quad (14)$$

2.3. Modifications to the damping term in mass flux computation

Stability analyses of shock capturing schemes using simple cases [15,19,20] were reported in several past studies. While most offered insights into the problems, some have offered actual prescriptions. For example, Dumbser and coworkers [21] presented a technique to predict threshold upstream Mach number for triggering odd–even instability for schemes with differential numerical fluxes. Their analysis also pointed to shock upstream region as the origin of the instability thus settling the debate between two contrary views [22,23]. Pandolfi and Ambrosio [19] analyzed many Riemann solvers including some from AUSM family and prescribed how to localize damping to cure carbuncle phenomena. Earlier work by Gressier et al. [15], using linear stability analysis, has shown that strict stability and exact contact line resolution are