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Extending the traditional Jeffery-Hamel flow to stretchable convergent/divergent channels

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ABSTRACT

The classical Jeffery-Hamel flow due to a point source or sink in convergent/divergent channels is extended in this paper for the first time in the literature to the case where the stationary channel walls are permitted to stretch or shrink. Such a physical mechanism is characterised by means of a parameter in the wall boundary conditions of the governing nonlinear differential equation. Results show that the classical flow and heat features are considerably altered by the application of sufficient stretching/shrinking of the walls. Stretching of the convergent or divergent channel is found to amplify the velocity profiles with an opposite effect in the case of shrinking resulting in back flow regions. As far as the temperature field is concerned, stretching leads to production of more heat in the flow, however, thermal layer is lowered and cooling is achieved by the presence of channel shrinking, which might have significant technological consequences.

1. Introduction

The two dimensional flow of a viscous, incompressible fluid between converging/diverging channels separated by a prescribed angle and driven by a line-source or sink at the apex is known as the traditional Jeffery-Hamel flow after pioneered research by Jeffery [1] and Hamel [2]. It has important applications particularly in fluid mechanics, chemical, mechanical and bio-mechanical engineering. A few precise applications are chemical vapor deposition reactors [3], high-current arc in plasma generators [4], expanding or contracting regions in industrial machines [5], gas compressors [6] and pipe sections [7]. Since Jeffery-Hamel flow constitutes one of rare exact solutions to the NavierStokes equations, it has been much attracted by the researchers in the literature specifically for validating different numerical approaches. The purpose of the present paper is to extent the classical Jeffery-Hamel flow problem to the cases, for the first time in the literature, where the convergent/divergent channels are subjected to stretching or shrinking, which might have important implications in science and industry.

Early studies on the traditional Jeffery-Hamel flow were done by Rosenhead [8]. The solution was expressed in terms of Jacobian elliptic functions. Exact solutions of the energy equation in the case of heat transfer in Jeffery Hamel flow were found by Millsaps and Pohlhausen [9] and Riley [10]. Fraenkel showed the existence of multiple solutions in general asymmetric case [11]. Terrill [12] investigated slow laminar flow in a converging or diverging channel with suction at one wall and blowing at the other wall. The textbook by Schlichting [13] also discussed the problem to some extend. Steady two dimensional incompressible laminar visco-elastic flow in a converging or diverging channel with suction and injection was studied by Roy and Nayak [14]. Three dimensional extensions to Jeffery-Hamel flow were thoroughly discussed and implemented in [15]. Experiments were also performed to understand the physical mechanisms behind the Jeffery-Hamel flow and its hydrodynamic stability features, see for instance Deshler [5], Putkaradze et al. [16] and Putkaradze and Vorobieff [17]. Magnetohydrodynamic effects of Jeffery-Hamel flow were recently explored numerically by Layek et al. [18] and Alam et al. [19].

Several practical applications in engineering and industrial processes, a few to cite: extrusion of polymer sheets from a die, drawing of plastic films, polyester thin wall heat shrink tubing, shrink film, wire drawing, glass fiber and paper production made the problem of stretching/shrinking surfaces very important, see for instance [20,21]. The pioneering work in this era is attributed by Crane [22]. After the experimental work by Tsou et al. [23], thousands of numerical or analytical works were followed, refer to the richness of the problem in the review paper by Wang [24]. Van Gorder et al. [25] considered the axisymmetric flow between two infinite stretching disks. Turkyilmazoglu [26–29] in a series of papers disclosed the effects of linear and exponential radial

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Nomenclature

Roman symbols

c_p	specific heat ($\mathrm{Jkg^{-1}\,K^{-1}}$)
C	stretching/shrinking parameter
c_f	coefficient of skin-friction
Ec	Eckert number
f	a function of θ
F	dimensionless radial velocity
g	a function of r
Nu	Nusselt number
p	pressure (Pa)
Pr	Prandtl number
Re	Reynolds number
q_w	heat flux ($\mathrm{Wm^{-2}}$)
r	radial direction in cylindrical polar coordinates
s	stretching/shrinking rate ($\mathrm{m^2s^{-1}}$)
T	temperature (K)
T_w	surface temperature of channels (K)
u	velocity component in the radial direction ($\mathrm{ms^{-1}}$)
u_c	rate of movement in the radial direction ($\mathrm{m^2s^{-1}}$)
u_w	wall velocity component in the radial direction ($\mathrm{m\,s^{-1}}$)

Greek symbols

α	elevation angle of convergent/divergent channels
β	a constant
η	a scaled coordinate
θ	azimuthal direction in cylindrical polar coordinates
Θ	scaled temperature
κ	thermal conductivity ($\mathrm{Wm^{-1}\,K^{-1}}$)
λ	a constant
μ	viscosity (Pas)
ν	kinematic viscosity ($\mathrm{m^2\,s^{-1}}$)
ρ	density ($\mathrm{kgm^{-3}}$)
τ_w	shear stress on the wall

stretching in magnetohydrodynamic rotating disk flows in both inertial and rotating frame of references.

Although the classical Jeffery-Hamel flow is well-documented in the literature, the mechanism of stretching/shrinking of the convergent/divergent channels has not been explored yet. It is believed that such a mechanism has physical grounds, see for instance the statement of Usta et al. [30] "Recent numerical and theoretical investigations have suggested that polymers migrate toward the centerline when hydrodynamic interactions are included, but our simulations show that in sufficiently narrow channels there is a reversal of direction and the polymers move toward the wall". Motivated by this fact, the current work aims to analyze this physical model. The consequences of stretching/shrinking of the channel walls on both the flow and temperature fields are highlighted by numerically solving the governing nonlinear differential equations, and also by detecting a few analytical solutions in some particular cases. Stretching is found to enhance the heat, whereas adequate cooling is possible when shrinking is accounted for.

The strategy employed in the rest of the paper is as follows. The governing equations, the leading nonlinear equations and exact solutions for some specific cases are obtained in Section 2. Implications of the results of the studied model are discussed in Section 3. Conclusions are eventually drawn in Section 4.

2. Problem definition

A class of exact solutions of the Navier–Stokes equations is due to the classical Jeffery-Hamel flow problem [13]. This flow is constructed in such a manner as the family of straight lines passing through a point in a plane constitute the streamlines of the flow. Hence, velocity differs from line to line so that it is a function of the polar angle θ. The rays along which the velocity identically vanishes can be regarded as the solid walls of a convergent or a divergent channel. As a result, the continuity equation can be satisfied with the assumption that the velocity along every ray is inversely proportional to the distance r from the origin.

Likewise, in the present case we consider the flow from a source/sink at the intersection between two stretchable or shrinkable walls coinciding at an angle 2α as sketched in Fig. 1. The walls are supposed to radially stretch or shrink in accordance with

$$u = u_w = \frac{s}{r},$$

with s being the stretching/shrinking rate. In the case $\alpha > 0$ walls considered are divergent, while $\alpha < 0$ represents convergent channel. As in the classical case, the velocity field is only along radial direction and depends only on r and θ so that $\mathbf{u} = (u(r,\theta), 0)$. The continuity equation, the Navier–Stokes equations and the energy equation governing the flow in polar coordinates (r,θ) are

$$\frac{\rho}{r}\frac{\partial(ru)}{\partial r} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} - \frac{u}{r^2}\right], \tag{2}$$

$$0 = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \frac{2\nu}{r^2}\frac{\partial u}{\partial \theta}, \tag{3}$$

$$u\frac{\partial T}{\partial r} = \frac{\kappa}{\rho c_p}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right] + \frac{\nu}{c_p}\left[2\left(\left(\frac{\partial u}{\partial r}\right)^2 + (ur)^2\right) + \left(\frac{1}{r}\frac{\partial u}{\partial \theta}\right)^2\right], \tag{4}$$

where p is the fluid pressure, T is the fluid temperature, ρ is the fluid density, ν is the coefficient of kinematic viscosity, κ is the thermal conductivity and c_p is the specific heat at constant pressure. The governing equations are accompanied with the boundary conditions, due to the symmetry assumption at the channel centerline $\theta = 0$

$$\frac{\partial u}{\partial \theta} = 0 = \frac{\partial T}{\partial \theta}, \quad u = \frac{u_c}{r}, \tag{5}$$

and due to the stretching/shrinking convergent/divergent wall condition at the plates $\theta = \pm\alpha$

$$u = u_w = \frac{s}{r}, \quad T = \frac{T_w}{r^2}, \tag{6}$$

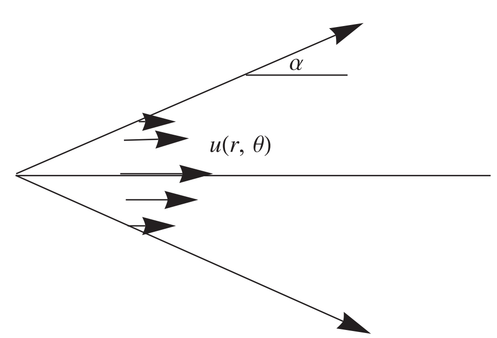

Fig. 1. Configuration of the flow and geometrical coordinates for a stretching divergent channel.