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An overlapping domain technique coupling spectral and finite elements for fluid flow

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ABSTRACT

Spectral elements are preferred for accurate prediction of transitional flows, whereas finite elements are better suited for, possibly non-smooth, deforming solid objects. A new overlapping domain technique is proposed that couples spectral and finite elements. This technique aims to efficiently capture the pressure jump and deformation rates of thin solids moving in a fluid. The solid object is fully embedded in a finite element fluid mesh and the coupling interface is situated in the fluid domain. The technique is particularly suited for simulation of cardiovascular fluid–structure interaction problems, such as flow through heart valves. In this paper the overlapping domain technique will be explained and several numerical benchmarks will be presented to test the fluid–fluid coupling between spectral and finite elements. The results obtained demonstrate that the technique is accurate and stable.

1. Introduction

Computations of transitional flow, a mixture of laminar and turbulent flow, at intermediate Reynolds numbers ($1000 < Re < 3000$) using the finite element method (FEM) require a large number of degrees of freedom to capture the small scale flow fluctuations. A higher-order spectral element method (SEM) is able to resolve such fluctuations using fewer degrees of freedom than FEM. Due to the less compact approximation, spectral elements are less suitable for describing non-smooth structures within elements, i.e. structures having a discontinuous slope at one or more points on the boundary. Especially, in Lagrangian formulations where geometrical changes have a relatively small length scale and solid deformations are large. These characteristics are particularly relevant for cardiovascular fluid–structure interaction (FSI) problems, like flow in the aorta distal to moving aortic valves. Based on these experiences an FSI method is proposed that accurately computes the small scale fluctuations of transitional flow using spectral elements and the stresses near as well as inside the strongly deforming elastic solid using finite elements.

In the context of this paper, FSI methods can be divided into three different groups: methods with conforming boundaries (ALE methods [1] and fictitious domain method with adaptive remeshing [2]), methods with non-conforming boundaries (immersed boundary method [3], fictitious domain method [4] and XFEM [5]) and methods with overlapping domains (Chimera [6] and ALE-Chimera techniques [7]). First, a short overview of these methods is given.

The Arbitrary Lagrangian–Eulerian (ALE) method has been developed to combine the advantages of the Lagrangian description, mainly used in structural mechanics, and the Eulerian description, frequently used in fluid dynamics. The nodes of the computational mesh are allowed to move in an arbitrary way, where the displacement of these nodes is taken into account in the convective term of the fluid equations [1,8–10]. ALE methods have a conforming fluid–structure interface. A Lagrangian description is applied to the fluid points on this fluid–structure interface and the grid deformation is, depending on the implementation, (partly) extended into the fluid domain. This strong fluid–solid coupling leads to accurate computations of the velocity, pressure and the interface location [11]. However, if the mesh becomes highly distorted due to large translations and/or rotations the method may become inaccurate. Computationally expensive remeshing is necessary and interpolation errors may be introduced.

The immersed boundary method is one of the first FSI methods using non-conforming boundaries. It was developed by Peskin in a finite difference context to investigate the flow in the heart and around heart valves [3,12–14]. Once again the (elastic) solid mesh

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is described in a Lagrangian way and the fluid in an Eulerian way, however, the solid mesh is free to move on top of the fluid mesh. Local body forces are added to the system of equations in order to impose the kinematic constraint between the solid boundary points and the interpolated fluid velocities at the same position.

A similar method, developed in a weighted residual finite elements context, is the fictitious domain method [4,15]. Lagrange multipliers are used to couple the fluid and the solid at the interface. In contrast to the immersed boundary method, the local body forces (Lagrange multipliers) are applied in the weak form of the momentum equations resulting in a distribution of the forces.

The immersed boundary and the fictitious domain method have non-conforming boundaries, where the solid boundary does not align with the edges of the fluid elements. This leads to inaccurate computations of the velocity (gradients) and pressure [2,16]. To circumvent this problem, van Loon et.al. developed an adaptive fictitious domain method with local remeshing to align the fluid elements with the solid boundary [2].

The immersed boundary, fictitious domain and adaptive fictitious domain method have a ‘fictitious’ fluid domain underneath the (elastic) solid domain, which has no physical relevance for the FSI-problem under consideration. For volume occupying structures, the physics of the structural domain is affected by the ‘fictitious’ fluid domain resulting in constraints for the structural deformation and leading to artificial viscosity and incompressibility [17].

To capture locally non-smooth properties, such as discontinuities, the extended finite element method (XFEM) has been developed [5,18]. In XFEM enrichment functions are added to the polynomial approximation space of the finite element method in the neighborhood of a discontinuity. Currently, XFEM is applied in a wide range of problems for structures and fluids [19]. One of the applications is the description of discontinuities in the velocity and stress computations for FSI with non-conforming boundaries by level set functions [20,21]. The advantages of XFEM are the removal of the fictitious fluid domain and the flexibility in choosing independently the mesh size of the fluid and the structure domain [17]. However, numerically the partially integrated elements can become very small resulting in an ill-conditioned matrix. Another disadvantage is that the partially integrated elements require a special adaptive integration scheme. This method is difficult to combine with SEM due to the loss of the tensor product formulation. Spectral elements describe larger areas than finite elements and, therefore, require more accurate integration techniques on a relatively large area to be described with XFEM enrichment functions.

Chimera methods can be characterized by the decomposition of the fluid domain into overlapping subdomains. They have been developed to simplify mesh generation for FSI-problems [6,22]. The entire fluid domain is covered by a structured background mesh. Depending on the position of the (moving) structure on top of the background mesh, nodes are deactivated, a process that is also called hole cutting [23]. This results in (at least) two interfaces, the first is the fluid–fluid interface and the second is the interface that remains, after removing the nodes in the structured background mesh. For the last interface, the classical Chimera method iteratively prescribes Dirichlet boundary conditions which are derived by interpolation of the previous solution. While for the fluid–fluid interface Dirichlet, Neumann or Robin boundary conditions can be prescribed [7,23]. A sequential iteration over the overlapping subdomains is performed to obtain a converged solution on the whole fluid domain.

The advantage of the Chimera method is its flexibility for computing moving bodies in fluids using a boundary fitted mesh between the fluid and (moving) solid. It allows the generation of independent meshes consisting of different element types, local mesh refinement and/or a different orientation of the elements [23]. Gamnitzer and Wall developed the ALE-Chimera method to compute fluid–structure interaction for (large) deformations of flexible structures in a fluid [7]. The disadvantages of the iterative fluid–fluid coupling are the increased computational costs and a loss of accuracy because of the velocity interpolation [17].

The goal of this study is to propose a new overlapping domain technique to couple spectral and finite elements in a monolithic FSI-method, which solves the fluid and solid equations simultaneously using one system matrix. The developed technique is based on the ideas of the Chimera method. The spectral element method is able to obtain accurate flow computations for transitional flows [24,25], whereas finite element methods are better suited for describing the possibly non-smooth elastic deformation of an elastic solid. The technique proposed aims to efficiently capture the pressure jump and deformation rates of thin solids moving in a fluid. In this overlapping domain technique, an additional fluid layer consisting of finite elements is conformally coupled to the elastic solid described by finite elements. This implies that the coupling between the finite element and the spectral element approximation moves to the fluid domain. Hence, a special fluid–fluid coupling is needed which will be described in this paper together with benchmark computations to demonstrate that the fluid–fluid coupling is accurate and stable.

This paper is structured as follows. Section 2 introduces the governing equations and Section 3 presents and explains the overlapping domain technique coupling SEM and FEM. Section 4 shows the results of different numerical test computations to compare SEM with FEM and to demonstrate the accuracy and stability of the proposed fluid–fluid coupling between SEM and FEM. Subsequently, the obtained results are evaluated and discussed in Section 5. Finally, Section 6 gives the main conclusions and directions for future work.

2. Governing equations

In this study fluid problems with a fluid–structure interface Γ_{fs} are considered, see Fig. 1, where the structure is a fixed rigid solid Ω_s. The fluid can be described by the incompressible Navier–Stokes equations in an Eulerian formulation

$$\rho\frac{\partial \boldsymbol{v}}{\partial t} + \rho\boldsymbol{v}\cdot\nabla\boldsymbol{v} = \nabla\cdot\boldsymbol{\sigma} + \boldsymbol{f}, \quad (1)$$

$$\nabla\cdot\boldsymbol{v} = 0, \quad (2)$$

for the fluid domain Ω_f. In these equations $\boldsymbol{v}$ is the fluid velocity, ρ is the fluid density, $\boldsymbol{f}$ a volumetric body force and $\boldsymbol{\sigma}$ the Cauchy stress tensor which is given by the constitutive equation for a Newtonian fluid

$$\boldsymbol{\sigma} = -pI + 2\eta\boldsymbol{D}(\boldsymbol{v}), \quad (3)$$

where p is the pressure, η the dynamic viscosity and $\boldsymbol{D}$ the rate of deformation tensor

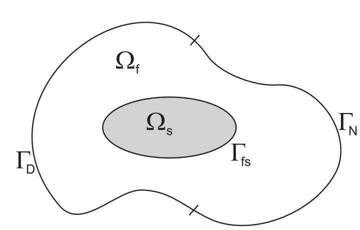

Fig. 1. A general visualization of the fluid domain Ω_f and the fixed solid domain Ω_s, with Γ_{fs} the boundary between the fluid and the structure, Γ_D and Γ_N the boundaries where respectively the Dirichlet and Neumann boundary conditions are imposed.