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Volume-conserving mesh smoothing for front-tracking methods

A. Toutant [a,b,c,*], B. Mathieu [c], O. Lebaigue [c]

[a] PROMES-CNRS, UPR 8521, Perpignan, France
[b] Université de Perpignan Via Domitia, France
[c] STMF/LMSF, CEA-Grenoble, Grenoble, France

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ABSTRACT

Among the various direct numerical simulation (DNS) methods dedicated to multiphase flow, the front-tracking methods that use a Lagrangian mesh to describe explicitly the interfaces are generally considered as a very accurate and complex method. In this family of methods, while a fine Lagrangian mesh is desirable for a better representation of the interfacial area, the surface forces and the bubble or droplet volume, one cannot arbitrarily choose the Lagrangian mesh size. Indeed, the Lagrangian mesh displacement algorithm is unstable if the number of Lagrangian degrees of freedom does not match the number of involved Eulerian velocity points. As a consequence, in traditional front-tracking implementations, an accurate description of the interfaces is expensive in terms of Eulerian mesh cells. We demonstrate that a front-tracking interface smoothing (FTIS) method can reduce the constraints on the mesh sizes. It consists in damping the highest spatial frequency components of the Lagrangian mesh to compensate for the lack of Eulerian velocity points. The test case of fundamental proper frequency of a bubble proves the validity of the FTIS method. An example of a 3D-bubble rising shows the interest and the potential applications of the FTIS method.

1. Introduction

Two phase flows are very common flows in natural environment and industrial processes. The direct numerical simulation (DNS) of interfaces is interesting to understand micro-level phenomena (e.g. mass or heat transfers at the interfaces, such as in [1]) that potentially drive macro-level phenomena (e.g. global efficiency of a heat exchanger). One difficulty to simulate interfaces is the discretization of surface tension. This force is proportional to the mean curvature of the interface, which is a second order derivative of the surface geometry. Among the DNS methods, we can distinguish the moving mesh methods, the front-tracking methods and the fixed mesh methods. The moving mesh methods consist in using a moving mesh where a surface tracks the shape of the interfaces. These methods are very efficient when bubbles are spherical [2,3]. Among these methods, one finds the ALE methods (Arbitrary Lagrangian Eulerian) [4]. The front-tracking methods use a fixed mesh for the volume variables and a moving surface mesh to represent the position of the interfaces [5,6]. This paper is dedicated to this kind of methods. The fixed mesh methods only use an Eulerian mesh. The balance equations of fluids and interfaces are solved with this fixed mesh. The interface capturing methods are the volume of fluid (VOF) methods [7,8] or the level-set methods [8]. The VOF methods use a phase indicator equal to 1 in one phase and to 0 in the other one. The level-set methods use a function where the interface position is implicitly defined by a level-set value of the function (e.g. the signed distance at the interface).

In the front-tracking methods, the interface is represented by a Lagrangian surface mesh. In some front-tracking implementations, high order polynomial elements are used to represent the interfaces in order to obtain good approximations of the curvature and of the interface position during the remeshing steps. See for example [10–12] where cubic splines are used. These implementations provides a reasonable accuracy with a relatively low Lagrangian markers density, but interpolation steps and exact volume computation of the phase volume and surface energy are quite difficult to implement [13]. In this paper, we present an implementation of a front-tracking based method where simpler plane surface elements are used to describe the interface. In this case, the exact calculation of the volume fraction and of the interface surface is much easier. These elements are then used to provide an exact discrete mass balance of the phases (including phase change), and a control of the discrete surface energy, which is strongly related to spurious currents.

The drawback of these low order surface elements is the lack of precision when only a few Lagrangian markers are used, which is normally the case because of the earlier mentioned stability issues. This paper describes a simple algorithm component to allow for a higher marker density in the simulations: the front-tracking

* Corresponding author at: PROMES-CNRS, UPR 8521, Perpignan, France.
E-mail address: adrien.toutant@univ-perp.fr (A. Toutant).

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interface smoothing (FTIS). It is worth noting that although front-tracking, VOF and level-set methods are originally DNS methods, several recent works try to extend these methods to some kind of Large Eddy Simulation (LES) for two-phase flows [14–19]. The FTIS method is especially useful in that context.

In Section 2, we examine the interest of the FTIS. Then, we describe the algorithm and properties. Finally, Section 3 is dedicated to the validation of the FTIS. Three test cases are realized. The first one concerns the oscillations of a 2D elliptical bubble. The second one verifies that the equilibrium of a gas bubble without any parasitic currents can be reached. The third one compares the terminal velocity and the bubble shape obtained with the FTIS method and an ALE reference simulation.

2. Front-tracking interface smoothing (FTIS)

2.1. Influence of the Lagrangian markers density on the stability and the accuracy of the front-tracking method

Since only linear elements are used to describe the interface, the computed volume and surface of any discretized interface geometry converges to the exact value with second order accuracy with respect to the Lagrangian markers spacing. Because buoyancy forces depend on the volume and viscous forces depend on the surface, the density of Lagrangian markers directly affects the accuracy of the simulations.

For example, let us consider a unity radius circle discretized with N Lagrangian markers. The area A and perimeter P of this discretized two dimensional bubble are:

$$A = N sin\left(\frac{\pi}{N}\right) cos\left(\frac{\pi}{N}\right) \quad (1a)$$

$$P = 2N sin\left(\frac{\pi}{N}\right) \quad (1b)$$

The real area is π and the real perimeter is 2π. Let x be the ratio $\frac{\pi}{N}$. The relative errors committed on the area E_A and on the perimeter E_P are:

$$E_A(x) = 1 - cos(x)\frac{sin(x)}{x} \quad (2a)$$

$$E_P(x) = 1 - \frac{sin(x)}{x} \quad (2b)$$

Fig. 1 represents the discretization errors as a function of the number of Lagrangian markers in the 2D case. The error on the bubble volume is higher than the error on the surface. In order to obtain a volume error below 5%, we must have $x < 0.28$, which means that the distance between markers must be three times smaller than the local radius of curvature.

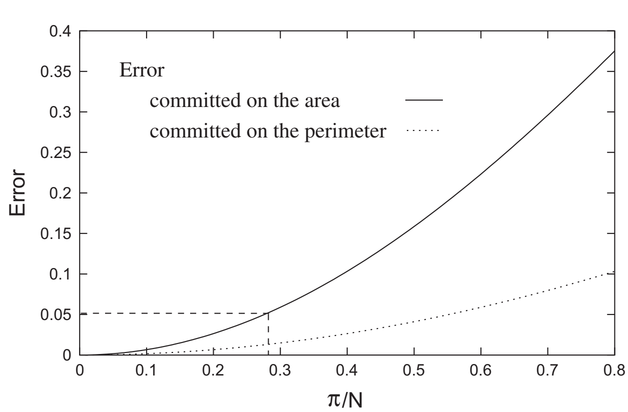

Fig. 1. Error coming from the discretization of the perimeter and impacting the value of the area of a circular interface in function of the number of Lagrangian markers N.

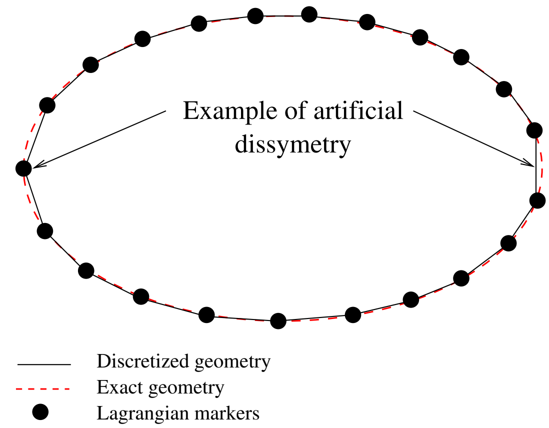

Fig. 2. Example of dissymmetry due to the discretization.

Furthermore, too few Lagrangian markers lead to an artificial asymmetry of the interface (see Fig. 2). The interfacial forces and the velocity field around the bubble are then asymmetric. The bubble trajectory could be directly affected by this numerical error [20].

Using a finer Lagrangian mesh is the simplest way to increase the accuracy of the method. Unfortunately, the traditional velocity interpolation used to update the Lagrangian mesh position is unstable if the Lagrangian mesh is too fine, as it will be shown in the next section.

2.2. Numerical instabilities related to the Lagrangian mesh refinement

In traditional implementations of the front-tracking method, the Lagrangian mesh position is updated by moving the mesh nodes with an interpolation of the Eulerian velocity field. One constraint for the numerical scheme to be stable, is that the interface shape can evolve towards an equilibrium position with a minimum of the surface energy (which, in the front-tracking method, is a Lagrangian mesh with the minimum surface, for a given phase volume). If the interface shape is not at mechanical equilibrium, the surface tension source term tends to accelerate the fluid in such a way that the surface energy will decrease (Fig. 3).

If the Lagrangian mesh node number exceeds the number of Eulerian velocity degrees of freedom involved in the displacement of these nodes, there exists a subspace of the set of all possible velocity fields of the Lagrangian mesh that cannot be generated by any interpolated Eulerian velocity field. This subset corresponds to high frequency oscillations of the surface mesh that produce no change of the discrete phase indicator function on the Eulerian mesh (Fig. 3). During the computation, these high spatial frequency oscillations will develop on the interfaces and no discrete Eulerian velocity field will be able to control them.

To sum up, the coupling between the Eulerian and the Lagrangian mesh quantities is effective only for spatial scales larger than the size of Eulerian cells. If a finer Lagrangian mesh is used, the physical damping of the high spatial frequencies on the Lagrangian mesh must be modelled by additional equations in the Lagrangian mesh transport equation.

2.3. The FTIS algorithm

The purpose of FTIS is to provide an appropriate damping of high frequency perturbations of the Lagrangian mesh not handled by the coupling with the Eulerian velocity field (see Fig. 4).

The main requirement for the damping model is that it provides the correct physical behavior at scales larger than the Eulerian