Computers & Fluids 66 (2012) 107–120

Contents lists available at SciVerse ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

A 3D pseudo-spectral low Mach-number solver for buoyancy driven flows with large temperature differences

Ouafa Bouloumou [a], Eric Serre [a,*], Patrick Bontoux [a], Jochen Fröhlich [b]

[a] M2P2 UMR 7340, CNRS – Aix-Marseille Université-Ecole Centrale, Technopole de Château-Gombert 38 rue F. Joliot-Curie, 13451 Marseille Cedex 20, France
[b] Institut für Strömungsmechanik, Technische Universität Dresden, 01062 Dresden, Germany

ARTICLE INFO

Article history:
Received 29 July 2011
Received in revised form 15 March 2012
Accepted 22 May 2012
Available online 20 June 2012

Keywords:
Spectral methods
Low Mach number approximation
Natural convection

ABSTRACT

A three-dimensional spectral method based on a Chebyshev/Chebyshev/Fourier discretization is proposed to integrate the Navier–Stokes equations for natural convection flow with large temperature differences under the low Mach-number approximation. The generalized Stokes problem arising from the time discretization by a second-order semi-implicit scheme is solved by a preconditioned iterative Uzawa algorithm. The spectrally convergent algorithm is validated on well-documented Boussinesq and non-Boussinesq benchmarks. Finally, non-Boussineq convection is investigated in a tall differentially heated cavity of aspect ratio 8 with one homogeneous direction. The technique is shown to be efficient in terms of computing performances and accuracy. The study brings new stability results evidencing 3D effects compared to both Boussinesq convection and former two-dimensional non-Boussinesq solutions. In particular, for $Ra \geqslant 10^5$ two-dimensional solutions are shown to be unstable with respect to three-dimensional perturbations. A Görtler-type instability grows in the region of the curved streamlines at both ends of the cavity and gives rise to 3D steady and oscillatory solutions. At higher Rayleigh numbers, these vortices eventually interact with a boundary layer instability leading to complex dynamics with multiple steady and unsteady stable solutions.

1. Introduction

Thermal convection in differentially heated enclosures has been of interest since a long time due to its fundamental importance in the perspective of transition to turbulence. Furthermore, it constitutes a prototypical configuration which is or relevance for many technological and geophysical processes. Typical engineering applications include nuclear reactors, solar energy collectors, electronic components in enclosures and others. High-fidelity experimental data on this topic are hard to obtain due to the difficulty in realizing boundary conditions with high precision, such as adiabatic conditions [1]. Hence, direct numerical simulations of such configurations are desirable since these allow to control all boundary conditions and modelling assumptions in detail. This provides a means to perform well-controlled physical studies.

Most simulations of natural convection are based on the Boussinesq (BO) approximation which is valid in the limit of small temperature differences [2]. When the vertical length scale is of the size of a few meters, which holds for most technical applications, this assumption is valid for temperature differences smaller than approximatively 30 K for gases such as air and smaller than a few degrees only for water. Here, we understand this approximation to imply that the density is set constant in all terms but the gravity term and that all other properties such as the expansion coefficient, the viscosity and the heat conductivity are set constant. Steady and unsteady convection developping in differentially heated enclosures in the BO regime have been investigated numerically for a long time since the pioneering work of [3]. A 2D benchmark problem was defined in [4]. Several numerical studies, and linear as well as weakly non-linear stability analysis were carried out showing intricate scenarios of transition and complex regimes. A relatively large literature exists on flows inside tall differentially heated cavities. An inverse transition from unstable to stable convection with increasing Rayleigh number was discovered by Roux et al. [5] and supported with a stability analysis by Brenier et al. [6]. In the low Prandtl number configuration, the onset of unsteady instabilities was investigated for melt flow applications with accurate description of the bifurcation detailing hysteresis cylcles and subcycles [7,8]. Convection regimes in cylindrical cavities were investigated by Le Quéré and Pécheux [9] in an axisymmetric annulus and by Crespo Del Arco and Bontoux [10] in a three-dimensional cylinder heated from below. For the same geometry but with the cylinder axis perpendicular to gravity, investigations were carried out to understand convection from crystal growth by vapor transport [11].

* Corresponding author. Tel.: +334 91 11 85 35; fax: +334 91 11 85 02.
E-mail address: eric.serre@L3m.univ-mrs.fr (E. Serre).

0045-7930/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compfluid.2012.05.014

Although the BO hypothesis allows to analyse important features, in many applications density (or temperature) differences are so large that non-Boussinesq conditions prevail. Non-Boussinesq effects in convection flows can arise from compressibility (density variations due to the pressure), density variations due to thermal expansion and variation of thermal conductivity or viscosity with temperature [12]. Variable thermodynamic properties can be incorporated in the numerical model fairly easily in most cases. On the contrary, density variations give rise to fast acoustic waves which are costly numerically. Indeed, since they propagate at a speed two or three orders of magnitude larger than the convective velocities, explicit time discretizations require very small time stepping to satisfy the CFL condition. Implicit discretizations lead to difficulties due to convergence problems of iterative solvers as the Jacobi matrix of the convective fluxes is poorly conditioned. Moreover, these waves are not of interest in most convection flows. Thus, only few studies mostly restricted to 2D flows were carried out in natural convection with a full compressible approach considering spectral methods [13,14] or finite volumes techniques [15–17].

A well-suited approach then is to devise special equations in which these acoustic waves are eliminated, the so-called low Mach number (LM) approximation of the Navier–Stokes equations (NSE) [18,19]. In these equations the pressure is split into a thermodynamic pressure which depends on time but is spatially homogenous, and a hydrodynamic pressure which ensures that the continuity equation is fulfiled. Only the thermodynamic pressure enters in the law of state. In contrast to the BO approximation, these equations allow to deal with arbitrary large density differences, provided the Mach number remains small, which is specified below. The velocity field then is no longer divergence free and the variation of density in space and time introduces supplementary non-linearities in the equations. Nevertheless, these equations have the same mixed hyperbolic-parabolic character as the equations for incompressible flow, so that similar numerical approaches can be employed.

The LM equations have been used in several studies of natural convection to investigate the effects of temperature differences larger than allowed by the BO approximation. For the Rayleigh-Bénard configuration, Fröhlich and Gauthier [14] demonstrated good agreement between the LM solution and fully compressible solutions since density variations are related to thermal expansion while compressibility effects remain weak. On the other hand LM solutions show large departures from the BO solutions with significantly smaller values of the critical Rayleigh number at which the instability is triggered [14]. Furthermore, in contrast to the BO approximation, the structure of the unsteady solutions shows a symmetry breaking between the hot and the cold boundary [12]. Using a van der Waals state equation Accary et al. [20] considered thermo-acoustic effects of the Rayleigh-Bénard instability problem in a cavity filled with a super-critical fluid. They performed detailed investigations of the onset of instabilities and discovered a reverse transition phenomenon. In tall differentially heated cavities, 2D studies [21,22] were concerned with the convection regimes obtained with different aspect ratios of the cavity and thermodynamic properties. A 2D benchmark for a square cavity was also proposed in [23].

Spectral methods perform extremely well for investigating transition to unsteadiness and turbulence due to their negligible numerical dissipation [24]. The high-order accuracy of these methods allows to determine the critical values of the bifurcation parameters with high precision and furthermore ensures a very accurate description of the unstable structures which are of weak intensity compared to the base flow. This has been demonstrated in particular for enclosed natural convection flows under the BO approximation, when steady and unsteady instabilities develop for a unicellular base flow at various Prandtl conditions [25,26,8,27]. For LM-convection, most of the numerical studies of closed cavities employing spectral methods have so far been limited to the two-dimensional (2D) case using either a Chebyshev/Fourier [28] or a Chebyshev/Chebyshev [21] expansion. In Fröhlich and Peyret [28], e.g., the use of a staggered grid requires repeated interpolations between the grids, and in [21] the derivative operators are tensorial and thus of large size.

Full 3D simulations using Chebyshev approximations lead to further algorithmic complexity. Algorithms based on Uzawa operator involve very large $N^3 \times N^3$ tensors that are computationally costly to invert and require large size memory. Fractional step method enhanced from BO convection and leading to a pressure equation could be more efficient [27]. Anyhow, the extension of these approaches to three dimensions is not straightforward. To the best of the authors' knowledge the LM convection equations have so far been solved in three dimensions only employing lower order numerical methods, typically second order [29] or at most fourth order in space using a fractional-step method [30].

The aim of the present work is to propose a robust and efficient pseudo-spectral algorithm to integrate NSE in the LM approximation applicable to air in a Cartesian enclosure with one direction of periodicity. This algorithm is then used to investigate the effect of a large temperature difference on the transition to unsteadiness for 2D and 3D flows in a tall differentially heated cavity of aspect ratio 8 with adiabatic top and bottom walls. The paper is laid out as follows. First, the governing equations are recalled. Next, the numerical algorithm is presented in detail and is validated with respect to an analytical solution. Spectral convergence of the error as well as the second-order accuracy in time are demonstrated. Then, the algorithm is physically validated by comparison with different benchmark solutions for the LM approximation in the case of a steady regime [23] and in the Boussinesq limit for an unsteady regime [4]. Finally, new results for the transition to 2D and 3D unsteady regimes are presented.

2. Modelling and problem formulation

In the following, the natural convection flow of air inside a differentially heated cavity is considered. The geometry is depicted in Fig. 1, where H is the height, L_y the width and L_x the depth of the enclosure, respectively. The aspect ratios of the cavity are defined as $A_y = H/L_y$ and $A_x = L_x/L_y$. The cavity has two horizontal adiabatic walls and one homogenous direction. The vertical walls in the z-direction are maintained isothermal at a hot and a cold temperature, T_h and T_c, respectively. Gravity is oriented in negative z-direction and the temperature difference $T_h - T_c$ is supposed

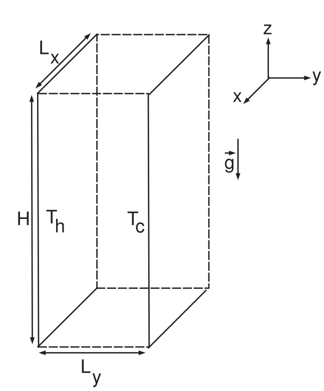

Fig. 1. Sketch of the differentially heated cavity. H is the height, L_y the width and L_x the depth of the enclosure in the homogeneous x-direction. The two horizontal passive walls are adiabatic and T_h and T_c are the hot and cold temperatures of the vertical walls. Gravity $\vec{g}$ is oriented in negative z-direction.