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Computation of stabilizing PI and PID controllers using the stability boundary locus

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Abstract

In this paper, a new method for the calculation of all stabilizing PI controllers is given. The proposed method is based on plotting the stability boundary locus in the (k_p, k_i)-plane and then computing the stabilizing values of the parameters of a PI controller. The technique presented does not require sweeping over the parameters and also does not need linear programming to solve a set of inequalities. Thus it offers several important advantages over existing results obtained in this direction. Beyond stabilization, the method is used to shift all poles to a shifted half plane that guarantees a specified settling time of response. Computation of stabilizing PI controllers which achieve user specified gain and phase margins is studied. It is shown via an example that the stabilizing region in the (k_p, k_i)-plane is not always a convex set. The proposed method is also used to design PID controllers. The limiting values of a PID controller which stabilize a given system are obtained in the (k_p, k_i)-plane, (k_p, k_d)-plane and (k_i, k_d)-plane. Furthermore, the proposed method is used to compute all the parameters of a PI controller which stabilize a control system with an interval plant family. Examples are given to show the benefits of the method presented.

Keywords: Stabilization; PI control; PID control; Gain and phase margins; Relative stabilization; Uncertain systems

1. Introduction

There has been a great amount of research work on the tuning of PI (proportional integral), PID (proportional integral derivative) and lag/lead controllers since these types of controllers have been widely used in industries for several decades (see Refs. [1,2] and references therein). However, many important results have been recently reported on computation of all stabilizing P (proportional), PI and PID controllers after the publication of work by Ho et al. [3–6]. A new and complete analytical solution based on the generalized version of the Hermite–Biehler theorem has been provided in Ref. [3] for computation of all stabilizing constant gain controllers for a given plant. A linear programming solution for characterizing all stabilizing PI and PID

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controllers for a given plant has been obtained in Refs. [4,6]. This approach, besides being computationally efficient, has revealed important structural properties of PI and PID controllers. For example, it was shown that for a fixed proportional gain, the set of stabilizing integral and derivative gains lie in a convex set. This method is very important since it can cope with systems that are open loop stable or unstable, minimum or non-minimum phase. However, the computation time for this approach increases in an exponential manner with the order of the system being considered. It also needs sweeping over the proportional gain to find all stabilizing PI and PID controllers, which is a disadvantage of the method. An alternative fast approach to this problem based on the use of the Nyquist plot has been given in Refs. [7,8]. A stability boundary locus approach for the design of PI and PID controllers has been given in Ref. [9]. A parameter space approach using the singular frequency concept has been given in Ref. [10] for design of robust PID controllers. More direct graphical approaches to this problem based on frequency response plots have been given in Refs. [11,12]. However, the requirement for frequency gridding has become the major problem for this approach.

Compensator design in classical control engineering is based on a plant with fixed parameters. In the real world, however, most practical system models are not known exactly, meaning that the system contains uncertainties. Much recent work on systems with uncertain parameters has been based on Kharitonov's result [13] on the stability of interval polynomials. Using the Kharitonov theorem, there have been many developments in the field of parametric robust control related to the stability and performance analysis of uncertain control systems represented as interval plants [14].

In this paper, a new approach is given for computation of stabilizing PI controllers in the parameter plane, (k_p, k_i) plane. The result of Ref. [8] is used to obtain the stability boundary locus over a possible smaller range of frequency. Thus, a very fast way of calculating the stabilizing values of PI controllers for a given SISO (single input, single output) control system is given. The proposed method is also used for computation of PI controllers for relative stabilization and for achieving user specified gain and phase margins. An extension of the method to find all stabilizing values of the parameters of a PID controller, namely k_p, k_i and k_d in the (k_p, k_i) plane, (k_p, k_d) plane and (k_i, k_d) plane, is also given. It is shown that the stability boundary of the convex polygon in the (k_i, k_d) plane for a fixed value of k_p can be generated from four straight lines. The equations of these straight lines can be easily derived using the stability boundary of the stabilizing regions obtained in the (k_p, k_i) plane and (k_p, k_d) plane. The proposed method is finally used for computation of PI controllers for the stabilization of interval systems.

The paper is organized as follows: The proposed method is described in Section 2. In Section 3, the computation of PI controllers for relative stabilization is given. The design of PI controllers that achieve user specified gain and phase margins is given in Section 4. Extension of the method to PID controllers is given in Section 5. In Section 6, the computation of PI controllers for interval plant stabilization is given. Concluding remarks are given in Section 7.

2. Stabilization using a PI controller

Consider the single input, single output (SISO) control system of Fig. 1 where

$$G(s) = \frac{N(s)}{D(s)} \tag{1}$$

is the plant to be controlled and $C(s)$ is a PI controller of the form

$$C(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s} \tag{2}$$

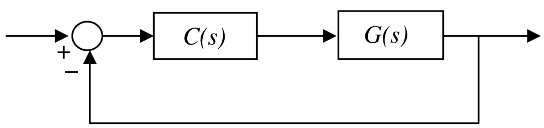

Fig. 1. A SISO control system.