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Cartesian grid method for the compressible Euler equations using simplified ghost point treatments at embedded boundaries

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ABSTRACT

We introduce two new approaches called the simplified and the modified simplified ghost point treatments for solving the 2D compressible Euler equations near embedded boundaries for the Cartesian grid method. These approaches are second order accurate for second order schemes near the embedded boundaries, if the wall boundary is in the middle between fluid and ghost points. We assign values to the ghost points near embedded solid boundaries from mirror points in the fluid to reflect the presence of the solid boundaries. In the simplified ghost point treatment, we consider the closest grid points on the grid lines through the ghost points in the *x*- and *y*-directions as the mirror points of the ghost points depending on which directions are closest to the directions normal to the embedded boundaries. In the modified simplified ghost point treatment, we choose mirror points not only on the grid lines through the ghost points in the *x*- or *y*-directions, but also on the diagonals through the ghost points. The primitive variables at the mirror points are mirrored to the ghost points using local symmetry boundary conditions. The simplified ghost point treatments at embedded boundaries are tested for supersonic flow over a circular arc airfoil and a circular cylinder. Applications to supersonic flow over multiple circular cylinders and a 2D model of the F-22 fighter aircraft demonstrate the flexibility of the ghost point treatments. Another advantage of these new approaches is that they are easily extendable to higher order methods and to 3D. The Cartesian grid method requires a larger number of grid points than the standard body-fitted grid method. We found a good agreement between the results obtained with the simplified and the modified simplified ghost point treatments and the reference solutions in the literature.

1. Introduction

The Cartesian grid method has recently become one of the widely used methods in CFD, cf. [1] for a review. This is due to its simplicity, faster grid generation, simpler programming, lower storage requirements, lower operation count, and easier post processing compared to body-fitted structured and unstructured grid methods. The Cartesian grid method is also advantageous in constructing higher order methods. Problems occur at the embedded boundary, when this method is applied to complex domains [2–4]. When the Cartesian grid method is applied at curved embedded boundaries, the cells at the embedded boundaries are not rectangular and these cut-cells create problems for the scheme to be implemented.

One method to solve the time step restriction problem caused by small cut-cells is to merge the cut-cells with neighboring cells [1,5]. The advantage of the cut-cell method is that it ensures conservativity, because a conservative method is used up to the embedded boundary where the normal velocity is set equal to zero. This approach requires a lot of effort to calculate the fluxes for all the cut-cells near the embedded boundary. Another approach to deal with irregular cells near the embedded boundary is to use the h-box method [6]. The basic idea of the h-box method is to calculate the fluxes at the faces of the small cells without reducing the time step determined by the stability condition for the Cartesian grid method with mesh size h. Apart from that, a new approach called Building-Cube Method has been introduced [7]. In this method, a high density mesh is proposed on the Cartesian grid. The wall boundary is approximated by a staircase and local Cartesian grid blocks called building cubes are used to capture the features in the boundary layer with high resolution. With the Building-Cube Method, complex geometries can be treated, adaptive grids can be used, and higher order schemes can be implemented. Recently, hybrid methods have been developed to treat embedded boundaries. The Cartesian grid method is combined with the gridless method near embedded boundaries [8–10].

Another approach is to use ghost points at the embedded boundary. In this method, symmetry conditions with respect to the embedded boundary are imposed at ghost points in the solid adjacent to the embedded boundary [11–13]. However,

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conservativity is lost in this process. Nevertheless, the simplicity of the ghost point treatment has motivated us to use the ghost point treatment instead of the more complicated approaches mentioned above.

The goal of the present study is to analyze the accuracy of the Cartesian grid method and to introduce new ghost point approaches called the simplified and the modified simplified ghost point treatments for the 2D compressible Euler equations. For spatial discretization, we use the local Lax–Friedrichs method and the MUSCL approach [14] with the minmod limiter. The first order explicit Euler and the third order TVD Runge–Kutta (RK3) methods are used for time integration. For the 2D compressible Euler equations, we compare the results of the new approach called the simplified ghost point treatment with the results of a body-fitted grid method for supersonic flow over a circular arc airfoil. Then, we compare our results with the results presented in the literature for supersonic flow over a circular cylinder [13]. The modified simplified ghost point treatment is applied to a moving shock wave over a circular cylinder. We compare the results of the modified simplified ghost point treatment with those by Luo et al. [8] obtained with a hybrid Cartesian grid and gridless method. The flexibility of the simplified and the modified simplified ghost point treatments is demonstrated for supersonic flow over three circular cylinders and a 2D model of the F-22 fighter aircraft.

The paper is organized as follows. In Section 2, the 2D compressible Euler equations and boundary conditions are introduced. An outline of the discretization techniques is given in Section 3. In Section 4, the flagging strategy and the simplified ghost point treatment at embedded boundaries are explained. The modified simplified ghost point treatment is introduced in Section 5. Results of applications to external aerodynamics are discussed in Section 6. In the end, conclusions are drawn in Section 7.

2. Compressible Euler equations

The 2D compressible Euler equations in conservative form read

$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}=0, \tag{1}$$

where

$$U=\begin{bmatrix}\rho\\ \rho u\\ \rho v\\ \rho E\end{bmatrix},\quad F=\begin{bmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ (\rho E+p)u\end{bmatrix},\quad G=\begin{bmatrix}\rho v\\ \rho uv\\ \rho v^2+p\\ (\rho E+p)v\end{bmatrix}, \tag{2}$$

with ρ, u, v, E, and p denoting density, velocity components in x- and y-directions, total energy per unit mass and pressure, respectively.

For perfect gas, we have the following relation

$$p=(\gamma-1)\left(\rho E-\frac{1}{2}\rho(u^2+v^2)\right), \tag{3}$$

where γ is the ratio of specific heats. We consider $\gamma = 1.4$ for air.

For supersonic flow in the x-direction, the conservative variables at the left boundary $x = x_a$ are given as Dirichlet boundary conditions $U(x_a,y,t) = g(y,t)$, cf. Fig. 1. No boundary conditions must be given at the right boundary $x = x_b$, because the flow is supersonic.

Symmetry boundary conditions at the symmetry boundary $y = 0$ imply:

$$(\rho,\rho u,\rho E)(x,y,t)=(\rho,\rho u,\rho E)(x,-y,t), \tag{4}$$

and

$$\rho v(x,y,t)=-\rho v(x,-y,t). \tag{5}$$

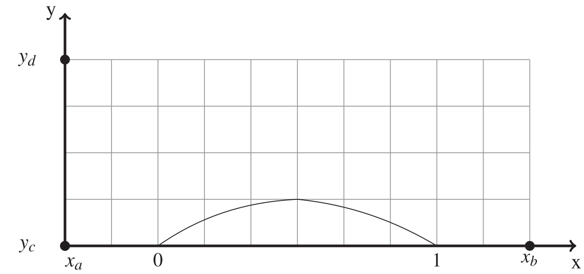

Fig. 1. Sketch of domain and Cartesian grid for supersonic flow over a circular arc airfoil.

Extrapolation boundary conditions are assumed at the upper boundary $y = y_d$:

$$\frac{\partial U}{\partial y}=0 \tag{6}$$

If the lower boundary is not a symmetry boundary, we also assume extrapolation boundary conditions (6) there.

3. Discretization

3.1. Spatial discretization

We assume a rectangular domain $[x_a,x_b]\times[y_c,y_d]$ and a $(I+1)\times(J+1)$ Cartesian grid with equidistant grid spacing $\Delta x = (x_b - x_a)/I$ and $\Delta y = (y_d - y_c)/J$. The Cartesian coordinates of the grid points (i,j) are (x_i,y_j), where $x_i = x_a + i\Delta x$, $i = 0, 1, \dots, I$ and $y_j = y_c + j\Delta y$, $j = 0, 1, \dots, J$.

The node-centered finite volume method yields the following semi-discretization of the 2D compressible Euler Eq. (1)

$$\frac{dU_{i,j}}{dt}=-\frac{F_{i+\frac{1}{2},j}-F_{i-\frac{1}{2},j}}{\Delta x}-\frac{G_{i,j+\frac{1}{2}}-G_{i,j-\frac{1}{2}}}{\Delta y}, \tag{7}$$

where $U_{i,j}$ is the approximation of the average of U in the cell $\Omega_{i,j}=\left[x_i-\frac{\Delta x}{2},x_i+\frac{\Delta x}{2}\right]\times\left[y_j-\frac{\Delta y}{2},y_j+\frac{\Delta y}{2}\right]$, i.e.

$$U_{i,j}\approx\frac{1}{\Delta x.\Delta y}\int_{\Omega_{i,j}}U(x,y,t)\quad dxdy \tag{8}$$

If we interpret (7) as a conservative finite difference method, $U_{i,j}$ is an approximation of the exact solution $U(x_i,y_j,t)$. $F_{i\pm\frac{1}{2},j}$ and $G_{i,j\pm\frac{1}{2}}$ are numerical fluxes for the 2D compressible Euler equations. The vector of the conservative variables U and the flux vectors F and G are defined in (2). The numerical fluxes of the local Lax–Friedrichs method for F and G are defined as follows

$$\begin{aligned}F^{lLF}_{i+\frac{1}{2},j}=\frac{1}{2}[&F(U_{i,j})+F(U_{i+1,j})-\max(|u_{i+1,j}|+c_{i+1,j},|u_{i,j}|+c_{i,j})\\&\times(U_{i+1,j}-U_{i,j})],\end{aligned} \tag{9}$$

$$\begin{aligned}G^{lLF}_{i,j+\frac{1}{2}}=\frac{1}{2}[&G(U_{i,j})+G(U_{i,j+1})-\max(|v_{i,j+1}|+c_{i,j+1},|v_{i,j}|+c_{i,j})\\&\times(U_{i,j+1}-U_{i,j})].\end{aligned} \tag{10}$$

In Eqs. (9) and (10), c is the speed of sound. The CFL number for the 2D compressible Euler equations is defined as $\mathrm{CFL}=\Delta t\max_{i,j}\left(\frac{|u_{i,j}|+c_{i,j}}{\Delta x}+\frac{|v_{i,j}|+c_{i,j}}{\Delta y}\right)$. We choose CFL = 0.5 in our numerical solutions of the 2D compressible Euler equations. In (9), we replace $U_{i,j}$ by $U^L_{i+\frac{1}{2},j}$ and $U_{i+1,j}$ by $U^R_{i+\frac{1}{2},j}$ using the MUSCL approach [14] with the minmod limiter to obtain higher order accuracy and also to avoid undesired oscillations. The extrapolated variables are defined as