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# Double-diffusive and Soret-induced convection in a micropolar fluid layer

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ABSTRACT

This paper reports an analytical and numerical study of natural convection in a shallow rectangular cavity filled with a micropolar fluid. Neumann boundary conditions for temperature and concentration are applied to the horizontal walls of the enclosure, while the two vertical ones are assumed insulated. The governing parameters for the problem are the thermal Rayleigh number, Ra, Prandtl number, Pr, Lewis number Le, buoyancy ratio, $\varphi$, aspect ratio of the cavity, $A$, and various material parameters of the micropolar fluid, $K$, $B$, $\lambda$ and $n$. For convection in an infinite layer ($A \gg 1$), analytical solutions for the stream function, temperature, concentration and microrotation are obtained using a parallel flow approximation in the core region of the cavity and an integral form of the energy and constituent equations. The critical Rayleigh numbers for the onset of supercritical and subcritical convection are predicted explicitly by the present model. Also, results are obtained from the analytical model for finite amplitude convection for which the flow and heat transfer are presented in terms of the governing parameters of the problem. Numerical solutions of the full governing equations are reported for a wide range of the governing parameters. A good agreement is observed between the analytical model and the numerical simulations. The influence of the material parameters on the flow and heat and solute transfers is demonstrated to be significant.



## 1. Introduction

Micropolar fluid theories were introduced by Eringen (see for instance Eringen [1–4]), in order to deal with a class of fluids which do not satisfy the Navier–Stokes model. Examples of applications of such fluids include the behavior of colloidal suspensions or polymeric fluids (Hudimoto and Tokuoka [5]), liquid crystals (Lockwood et al. [6]); animal blood (Ariman et al. [7]), exotic lubricants (Eringen [4]), etc. An excellent review of the various applications of micropolar fluid mechanics was presented by Ariman et al. [8] and Ariman and Turk [9].

The onset of convective instabilities in a horizontal layer of a micropolar fluid heated from below has been considered first by Ahmadi [10]. A solution was obtained in the case of free boundaries and it was demonstrated that the micropolar fluids are more stable than the Newtonian one. The same problem was extended by Rama Rao [11] who studied the onset of convection of a heat conducting micropolar fluid layer confined between two horizontal rigid boundaries. The effects of non-uniform temperature profiles on Marangoni convection in micropolar fluids confined between a lower rigid isothermal boundary and an upper free, constant heat flux boundary was investigated by Rudraiah et al. [12]. It was also demonstrated by these authors that micropolar fluids heated from below are more stable when compared to the pure viscous fluid situation. More recently, Rayleigh-Bénard convection in a micropolar ferromagnetic fluid has been investigated analytically by Abraham [13] for a layer with free–free, isothermal, spin-vanishing magnetic boundaries. The influence of the micropolar parameters on convection in the ferromagnetic case is similar to its role in the non-magnetic fluid case. More recently, the effect of a non-uniform basic temperature gradient on the onset of Marangoni convection in a horizontal micropolar fluid layer was considered by Melviana et al. [14]. It was assumed that the layer is bounded below by a rigid plate and above by a nondeformable free surface subjected to a constant heat flux. At these boundaries the microrotation was assumed to be vanished. The influence of various parameters on the onset of convection is discussed. Also, a linear stability analysis was performed by Idris et al. [15] to study the effect of non-uniform basic temperature gradients on the onset of Benard–Marangoni convection in a micropolar fluid. The influence of various parameters on the onset of convection has been analysed by these authors. It was found that the presence of micron-sized suspended particles delays the onset of convection.

All the above studies are concerned with convective flows induced by thermal gradients solely. However, in practice micropolar fluids may additionally have salt dissolved in it such that there are potentially two destabilizing sources for the density difference. Depending on how the temperature and mass fraction gradients are oriented relative to one another, the dynamics of convection

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**Nomenclature**

| | |
|---|---|
| $A$ | aspect ratio of the cavity, $L'/H'$ |
| $a$ | integer number as $a = 0$ or 1 |
| $B$ | microinertia parameter, $H'^2/j$ |
| $C$ | mass fraction of the reference component |
| $D$ | isothermal diffusion coefficient |
| $D'$ | thermodiffusion coefficient |
| $C_S$ | dimensionless concentration gradient in $x$-direction |
| $C_T$ | dimensionless temperature gradient in $x$-direction |
| $g$ | gravitational acceleration |
| $H'$ | height of fluid layer |
| $j$ | microinertia per unit mass |
| $k$ | thermal conductivity |
| $K$ | vortex viscosity parameter, $\kappa/\mu$ |
| $L'$ | width of fluid layer |
| Le | Lewis number, $\alpha/D$ |
| $N$ | dimensionless microrotation, $N'H'^2/\alpha$ |
| Nu | Nusselt number, Eq. (34) |
| $n$ | dimensionless microgyration parameter |
| Pr | Prandtl number, $\nu/\alpha$ |
| $q'$ | constant heat flux per unit area |
| Ra | thermal Rayleigh number, $g\beta'_T\Delta T'H'^3/\alpha\nu$ |
| $Ra_C^{sub}$ | subcritical Rayleigh number, Eq. (32) |
| $Ra_C^{sup}$ | supercritical Rayleigh number, Eq. (31) |
| $S$ | normalized mass fraction, $C/\Delta C$ |
| Sh | Sherwood number, Eq. (35) |
| $t$ | dimensionless time, $t'\alpha/H'^2$ |
| $T$ | dimensionless temperature, $(T' - T'_0)/\Delta T'$ |
| $u$ | dimensionless velocity in $x$ direction, $(u'H'/\alpha)$ |
| $v$ | dimensionless velocity in $y$ direction, $(v'H'/\alpha)$ |
| $x$ | dimensionless coordinate axis, $(x'/H')$ |
| $y$ | dimensionless coordinate axis, $(y'/H')$ |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | thermal diffusivity |
| $\beta_C$ | concentration expansion coefficient |
| $\beta'_T$ | thermal expansion coefficient |
| $\mu$ | dynamic viscosity |
| $\nu$ | kinematic viscosity of fluid, $\mu/\rho$ |
| $\varphi$ | buoyancy ratio, $(\beta_C\Delta C/\beta'_T\Delta T')$ |
| $\lambda$ | material parameter, $\gamma/(\mu j)$ |
| $\rho$ | density of fluid |
| $\Psi$ | dimensionless stream function, $\Psi'/\alpha$ |
| $\gamma$ | spin gradient viscosity |
| $\kappa$ | vortex viscosity |

*Subscript*

| | |
|---|---|
| 0 | reference state |
| $c$ | refers to critical conditions |

*Superscript*

| | |
|---|---|
| $'$ | refers to dimensional variable |

in such fluids can be very different from those driven only by thermal buoyancy. Most studies available on the onset of convection in a micropolar fluid layer, induced by both thermal and solutal density gradients, are concerned with double diffusive convection. For this situation the flows induced by the buoyancy forces result from the imposition of both thermal and solutal boundary conditions on the horizontal boundaries of the layer. Sharma and Kumar [16] investigated the thermosolutal convection in a layer of electrically conducting micropolar fluids heated and salted from below in the presence of a uniform vertical magnetic field. It was found that coupling between thermosolutal and micropolar effects may bring overstability in the system. The same configuration was also considered by Sunil et al. [17], who derived an exact solution in the case of a fluid layer contained between two free boundaries. The influence of various parameters, like solute gradient and micropolar parameters on the onset of convection are reported by these authors. Another type of problems investigated in the field of natural convection in binary fluids is related to the phenomenon of thermal diffusion, also known as the Soret effect. For this situation, when a temperature gradient is applied to a binary mixture, initially homogeneous, thermal diffusion takes place, giving rise to a solutal gradient. To the authors' knowledge the only study related to the Soret effects on natural convection of a micropolar fluid, due to Rawat and Bhargawa [18], is concerned with the case of a vertical plate embedded in a Darcy porous medium. The influence of the Soret number on the problem was found to be significant. However, concerning the onset of convection in a horizontal layer of a micropolar fluid, no study on the influence of the Soret effects on the onset of convection does seem to have been carried out. This is the main motivation of the present investigation.

In the present paper, we consider natural convection in a horizontal layer of a micropolar fluid with the horizontal boundaries heated and salted from the bottom by constant fluxes (Neumann boundary conditions). The paper is organized as follows. In the next sections, the formulation of the problem is first presented. An approximate analytical solution is then derived. A numerical method of the full governing equations is proposed in the following section. Results are then presented and discussed. The last section contains some concluding remarks.

## 2. Mathematical formulation

The configuration considered in this study is a horizontal shallow cavity, of thickness $H'$ and width $L'$ filled with a micropolar fluid (see Fig. 1). The origin of the coordinate system is located at the center of the cavity with $x'$ and $y'$ being the horizontal and vertical coordinates, respectively. Neumann boundary conditions for temperature and concentration are applied to the horizontal walls of the enclosure, while the two vertical ones are assumed insulated. The fluid is assumed to satisfy the Boussinesq approximation,with constant properties except for the density variations in the buoyancy force term. The density variation with temperature and concentration is described by the state equation $\rho = \rho_0[1 - \beta'_T(T' - T'_0) - \beta_C(C - C_0)]$ where $\rho_0$ is the fluid mixture density at temperature $T' = T'_0$ and mass fraction $C = C_0$ and $\beta'_T$ and $\beta_C$ are the thermal and concentration expansion coefficients, respectively.

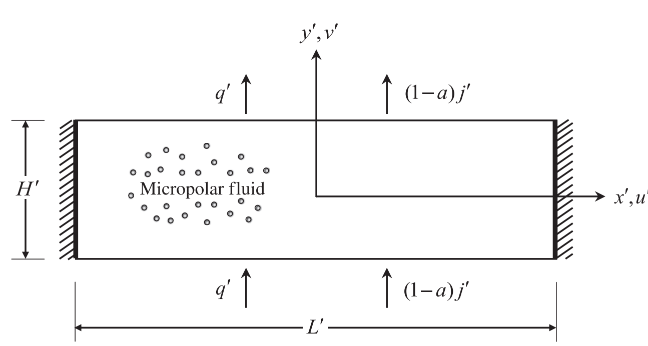


**Fig. 1.** Schematic diagram of the problem domain and coordinate system.