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On sensitivity of RANS simulations to uncertain turbulent inflow conditions

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ABSTRACT

The present study deals with gPC-based analysis of the sensitivity of URANS simulations to uncertain inflow conditions. The massively separated flow around a square cylinder, which is a classical test case, is selected. Three popular turbulence models are selected, i.e. standard low-Reynolds k–ε model, k–ω model and a realizable k–ε model. The turbulent viscosity ratio, which is observed to vary a lot according to different authors in this configuration, is taken as an uncertain parameter to illustrate the potential applications of the present methodology. It is found that the realizable k–ε model behaves quite stably, and predicts nearly consistent results in all simulations. In contrast, the other two models are very sensitive to random turbulent inflow condition in both cases, especially the standard k–ω model, resulting in a significant weakness of usual validation procedures for turbulence models.

1. Introduction

Numerical modelling and simulation is playing an essential role in studying complex physical systems. The key issue of successful simulation relies on the reliable mathematical model accounting for the essential characteristic of the system. However, the mathematical model often reflects an idealized situation that may not be achieved practically. In many cases, there are several uncertainties involved in the model, such as the input data set may not be completely specified due to incomplete knowledge of the real system. It is always the case in numerical simulation that turbulent inflow condition is incomplete especially when Reynolds-Averaged Navier–Stokes (RANS) turbulence model is employed. Thus, even though the model equations are deterministic, it may not be possible to rely on single deterministic simulation because the input data are not precisely known. Consequently, it is essential to associate the simulation results with the uncertainties, especially when the physical systems are sensitive to these uncertainties. This relies on the Uncertainty Quantification (UQ) in Computational Fluid Dynamics (CFD). The topic has received increasing attention in recent year [17]. With the help of UQ, additional statistical information can be produced from deterministic numerical methods and the outline of stochastic response of uncertainty in the system can be established.

The goal of the present paper is to illustrate the sensitivity of RANS simulations to uncertain parameters and the ability of gPC-based methods to give a deep insight into this sensitivity. A large sensitivity with respect to uncertain parameters, which are ubiquitious in complex configurations, may prevent validation of numerical methods or turbulence models in the traditional way, since discrepancies with reference data may stem from propagation and amplification of uncertainties and not from flaws of the method. The flow around a cylinder is selected as an example, since it has been studied by a large number of research groups via numerical simulation, including LES, RANS, URANS and hybrid RANS–LES approaches. A common issue deals with prescription of turbulent boundary conditions at the inlet plane. The problem stems from the fact that the features of incoming turbulence is not completely known from experimental studies. Practically, different arbitrary choices have been made by authors to prescribe inlet turbulence. The turbulent flow past a square cylinder at Re = 22,000 was experimentally investigated by Lyn et al. [21] and is a widely used test case. The free-stream turbulence intensity was measured and reported with a value of $I = 2\%$, i.e. the turbulent kinetic energy of the inflow can be determined precisely. However, none of the experimental data was reported concerning the dissipation rate of the turbulent kinetic energy (or integral length scale) of the inflow. In extensive simulations concerning this flow, the incomplete turbulent boundary condition is generally treated in such a way that a proper ratio ($r = \mu_t/\mu$) of the turbulent viscosity (μ_t) to dynamic viscosity (μ) is prescribed. Then the dissipation rate of kinetic energy at the inlet can be determined accompanied with the already known turbulent intensity. According to Lakehal and Thiele [13], the computations employing any of the low-Re isotropic models did not succeed on the basis of choosing $r > 50$. Compared with Franke and Rodi [4] who used a large value of $r = 100$,

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relative success was reported when opting small value of r, for example, $r = 7.92$ by Raisee et al. [23], $r = 10$ by Rodi [24], Bosch et al. [1], Saha et al. [25], Kimura et al. [11], and $r = 20$ by Kawamura et al. [10], Johansen et al. [6]. These studies confirm that the choice of the value of r has significant influence on the results. For a particular RANS model, although satisfactory results may be obtained if a proper value of r is imposed, the performance of this model for such flow is still unreliable as the predictions are sensitive to the value of r and its effects are not accounted. The finding motivates an analysis in the paper of the implications of the freestream turbulence in modelling this flow based on the RANS models. Here, the freestream turbulence is treated as random variable instead of a determined one. The sensitivity of the RANS simulation with respect to the random input is presented and the effects are quantified.

This uncertainty can be analyzed through UQ approach. There are several such approaches exist. Among the most used in the CFD framework, one can mention Kriging methods [2,16,7,8,15,9] and generalized Polynomial Chaos (gPC) [20,12,27,17,3,19,22,18]. The later is used in the present work.

2. Numerical model

The current investigations focus on the unsteady turbulent flow past a square cylinder at Re = 22,000 (based on the inlet velocity U_{in} and the length-side d) experimentally investigated by Lyn et al. [21]. The computational domain extends to $20d$ (with d the length-side of the square cylinder) downstream the centre of the square cylinder and $5d$ upstream. In the transverse direction, the computational domain is taken to be $14d$. A Cartesian nonuniform grid is used with about 65,000 cells, with grid refinement near the cylinder. The first grid neighbouring the wall is set with a distance of $\Delta y/d = 7.81 \times 10^{-4}$. The grid independence tests were accomplished with respect to the time-averaged values of the integral parameters such as drag and lift coefficients and Strouhal number. No-slip boundary conditions are used on solid walls. Governing equations are solved using a second-order accurate finite volume method. The QUICK scheme is used for convective terms.

The turbulence models employed for computations are the standard k–ε model with low-Re modification proposed by Launder and Scharma [14], the standard k–ω model by Wilcox [28] and a realizable k–ε model by Shih et al. [26]. For convenience, they are referred as $k\varepsilon$_std, $k\omega$ and $k\varepsilon$_rea model respectively in the following.

For the turbulent inflow condition, the turbulent intensity is taken equal to $I = 2\%$ as in experimental data, and the viscosity ratio $r = \mu_t/\mu$ is taken as a random variable with uniform distribution. The range of variation of r is taken equal to $[0,100]$, in accordance with values reported by previous authors. Once these two quantities are prescribed, the remaining ones can be obtained directly, for example turbulent kinetic energy $k = 1.5(IU_{in})^2$, dissipation rate $\varepsilon = C_\mu \rho k^2/r\mu$ and $\omega = \varepsilon/k = C_\mu \rho k/r\mu$, with $C_\mu = 0.09$.

The mathematical framework of gPC method will be briefly described here and more details are contained by Ghanem and Spanos [5]. The reader is also referred to recent papers for a detailed description of the present gPC implementation [20,12,27,19,22,18]. The polynominals of Askey scheme are used in gPC approach [30], more specifically, the Legendre polynominals are used as they turn optimal for a random variable with uniform distribution, as assumed for r in the present study. According to the gPC theory, one can use the following truncated polynomial expansion of the solution:

$$u(\vec{x};r) = \sum_{m=0}^{P} u_m(\vec{x})\phi_m(r), \qquad (1)$$

where P is the maximum polynominal order in the expansion and ϕ_m is the mth-order Legendre polynomial.

To evaluate the deterministic modal coefficient u_m, there exist so-called intrusive and non-intrusive approach. The non-intrusive method [29] is used. It is a stochastic collocation method. The solution is directly projected onto each member of the orthogonal basis chosen to span the random space. The projection coefficients are computed thanks to a Gauss–Legendre quadrature method. Once the coefficients have been computed, the response surface can be computed, along with the probability density function of the solution and all statistical moments (mean value, variance, ...).

Numerical tests have shown that $P = 6$ with 7 quadrature points were required to get a fully converged solution.

3. Results

There are some important integral parameters describing the oscillating unsteady flow, such as Strouhal number (St), drag and lift coefficients. The Strouhal number is a dimensionless number and defined as St = fd/U_{in}, where the primary vortex shedding frequency f is obtained by FFT of lift coefficient. Besides, there are two additional important mean flow parameters, i.e. the length of the recirculation bubble downstream the cylinder (l_r) and the maximum amplitude of negative velocity inside the recirculation bubble ($|U_r|_{max}$). The mean value and standard deviation of the mean flow parameters are summarized and compared in Table 1, where max. PDF refers to the value with maximum PDF, sta. mean is related to the mean statistical value (i.e. integral of the PDF), and c_v = standard deviation (STDD)/sta. mean is the coefficient of variation.

The results show that predictions by $k\varepsilon$_rea model are not sensitive to r, while significant variations are observed for the other two models, since there are big differences between the max. PDF value and the sta. mean value, especially for l_r, $|U_r|_{max}$, the rms drag and lift coefficients. This implies that the two models are sensitive to r, and moreover the results dealing with c_v demonstrate that the $k\omega$ model is much more sensitive than $k\varepsilon$_std model. It should be noted that $C_{D,mean}$ has relative small variance than the other parameters. As input r is a variable, the max. PDF value is much more meaningful than sta. mean value to represent the resolution of relevant turbulence model. The max. PDF results show that none of the

Table 1
Comparison of mean value and standard deviation of integral parameters, length of recirculation bubble and maximum amplitude of negative velocity in the bubble due to variable r.

	St	$C_{D,mean}$	$C_{D,rms}$	$C_{L,rms}$	l_r	$\lvert U_r\rvert_{max}$
$k\varepsilon$_std model						
Max. PDF	0.120	1.639	0.0007	0.049	3.465	0.153
Sta. mean	0.124	1.651	0.0008	0.114	3.390	0.174
STDD	0.009	0.037	0.0014	0.087	0.185	0.024
c_v(%)	7.385	2.239	174.6	76.1	5.468	13.80
$k\omega$ model						
Max. PDF	0.135	1.923	0.003	0.098	3.224	0.216
Sta. mean	0.124	1.973	0.025	0.600	2.337	0.184
STDD	0.012	0.091	0.060	0.611	0.935	0.061
c_v (%)	9.877	4.604	241.6	101.7	40.02	33.22
$k\varepsilon$_rea model						
Max. PDF	0.143	1.931	0.014	0.749	1.905	0.169
Sta. mean	0.142	1.936	0.014	0.755	1.897	0.169
STDD	0.001	0.007	0.0003	0.006	0.012	0.0002
c_v(%)	0.784	0.367	2.373	0.849	0.655	0.148
Ref.						
Std.k–ε [24]	0.134	1.64	≈0	0.305	2.8	–
LS k–ε [23]	0.115	1.99	–	–	2.3	–
Exp. [21]	0.132	2.10	–	–	1.38	0.21