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Proper orthogonal decomposition of a fully confined cubical differentially heated cavity flow at Rayleigh number $Ra = 10^9$

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ABSTRACT

Proper orthogonal decomposition is used to educe fundamental velocity and temperature coherent structures in the fully confined cubical three-dimensional differentially heated cavity (DHC) flow. Among other linear decompositions, the POD is optimal in the sense that it provides a set of modes that captures the largest amount of *energy* contained in the snapshot ensemble. We present here preliminary results of the first empirical eigenfunctions that account up to 95% of the total *energy* of the ensemble. The database is made of 200 snapshots obtained by means of Direct Numerical Simulation (DNS) at Rayleigh $Ra = 10^9$. The results are in good agreement with previous observations of coherent structures identified with λ_2 criterion, confirming the importance of the elongated spanwise structures (located downstream the break up of the laminar vertical boundary layers) for the description/modeling of the turbulent heat flux. The basis functions that account for the largest part of the turbulent heat flux are not made of the most energetic POD empirical eigenfunctions. In appears that the spatial structures which contain the largest fraction of the turbulent heat transfer correspond to the POD modes characterized by the presence of spanwise elongated vortices at the vertical active walls where temperature and velocity eigenfunctions are spatially strongly correlated.

1. Introduction

The identification of coherent structures in complex flows is of primary importance for understanding the underlying physics and for simplifying their dynamics. Among other identification methods, for instance conditional averaging, λ_2 *criterion* [1], or coherent vortex extraction (CVE) [2], the proper orthogonal decomposition (POD) [3] is a linear decomposition technique which provides a set of optimal spatial empirical eigenfunctions that are orthonormal (for a comparison between CVE and POD we refer to [4,5]). It captures the largest possible amount of *energy* (the concept of energy will be defined hereafter) for any given number of modes. These intrinsic features of the POD allow to get not only insight of the different fluid structures appearing in a collection of events, but also to sort them accordingly to their averaged *energetic* content when the eigenfunctions are projected on the original database. Finally, the empirical eigenfunctions can be used for simulating the interaction between themselves, i.e. the dynamics of the chaotic system, by performing Galerkin projection of the Navier–Stokes equations on the spatial structures. The latter is a powerful tool for flow control or instability analysis.

Natural convective flows are present not only in nature but also in several different industrial applications (e.g. chemical and nuclear reactors, building air conditioning, electronic cooling systems, etc.). Previous investigations of natural convective flows using POD can be found in [6–8]. These studies analyse the classical Rayleigh–Bénard problem. To the authors' knowledge, the only works dealing with POD in the differentially heated cavity flows in two-dimensional tall cavities may be found in [9,10] and in [11] for the three-dimensional case at very low Prandtl numbers. Moreover, only few published results focused on the three-dimensional fully confined supercritical flows in this configuration [12–14].

This work will present preliminary results of velocity and temperature empirical eigenfunctions of fully three-dimensional differentially heated cavity flow at Rayleigh number $Ra = 10^9$, the study of low-dimensional dynamical model will be the object of future investigations. In order to decrease the computational costs, Sirovich' snapshot method [15] is used on an ensemble of 200 statistically steady frames. Only the most energetic modes that account up to the 95% of the ensemble averaged fluctuating *energy* are retained. Results are in good agreement with previous observation of velocity coherent structures [12,16]. Furthermore, the combination of velocity and temperature modes will be investigated for modeling the turbulent heat flux.

The article is organized as follows: a brief description of the numerical method used for creating the DNS database is provided

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in Section 2; some fundamental aspects of the POD technique are given in Section 3; Section 4 reports the results of the empirical eigenfunctions for velocity and temperature, including the POD spectra, the cumulative energy distributions, modeling and physical interpretation of turbulent heat flux coherent structures; the last Section 5 is devoted to the concluding remarks and perspectives.

2. Direct numerical simulation

We consider the flow of air in a cubical cavity of edge-length H, open domain $\hat{\Omega} = (]-0.5H, +0.5H[)^3$ and $\partial\hat{\Omega}$ its boundary (Fig. 1). This flow is driven by the local differences of fluid density caused by the application of a temperature gradient between two opposite vertical walls: the hot wall is kept at a constant temperature T_H and the cold wall at T_C, whilst all the remaining walls are adiabatic. The temperature $\Delta T = T_H - T_C$ is supposed to be sufficiently small for the Boussinesq approximation to hold. Moreover the fluid is assumed homogeneous, incompressible, Newtonian with constant kinematic viscosity ν_r, constant thermal diffusivity κ_r and volumetric thermal expansion coefficient $\alpha_r = -1/\rho(\partial\rho_r/\partial T)_{p_r} = 1/T_r$ (the subscript r indexes quantities that are measured at the reference mean temperature $T_r = \frac{T_H+T_C}{2}$). Under these hypotheses momentum, mass and energy conservation read in dimensionless vector form as:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \frac{Pr}{\sqrt{Ra}}\Delta\mathbf{u} - Pr\Theta\frac{\mathbf{g}}{|\mathbf{g}|} \text{ in } \Omega, \quad (1)$$

$$\nabla\cdot\mathbf{u} = 0 \text{ in } \Omega, \quad (2)$$

$$\frac{\partial\Theta}{\partial t} + \mathbf{u}\cdot\nabla\Theta = \frac{1}{\sqrt{Ra}}\Delta\Theta \text{ in } \Omega, \quad (3)$$

with boundary conditions[1]

$$\mathbf{u} = \mathbf{0} \text{ on } \partial\Omega, \quad (4)$$

$$\Theta(x_1 = \pm 0.5, x_2, x_3) = \mp 0.5 \quad \text{for all } x_2,\ x_3 \in [-0.5, +0.5], \quad (5)$$

and

$$\frac{\partial\Theta}{\partial x_2}(x_1, x_2 = \pm 0.5, x_3) = 0 \quad \text{for all } x_1,\ x_3 \in [-0.5, +0.5], \quad (6)$$

$$\frac{\partial\Theta}{\partial x_3}(x_1, x_2, x_3 = \pm 0.5) = 0 \quad \text{for all } x_2,\ x_3 \in [-0.5, +0.5] \quad (7)$$

where the following scaling for the velocity, length, time and pressure are used [16–18]

$$U^* = \frac{\kappa_r}{H}\sqrt{Ra}, \quad L^* = H, \quad t^* = \frac{H^2}{\kappa_r\sqrt{Ra}}, \quad P^* = \rho_r U^{*2}. \quad (8)$$

The dimensionless unknowns are the velocity vector $\mathbf{u} \equiv (u_1, u_2, u_3)$, p the pressure and $\Theta = \frac{T-T_r}{\Delta T}$ the temperature difference. There are only two governing parameters, the Prandtl number $Pr = \nu_r/\kappa_r = 0.71$ (for air) and the Rayleigh number $Ra = g\alpha_r\Delta TH^3/(\nu_r\kappa_r)$ set to 10^9.

The spatial approximation of any field relies on the expansion in tensor product of Chebyshev polynomials of order $M = 169$ along every space direction and an usual collocation method is applied at the Chebyshev–Gauss–Lobatto points [19,20]. The projection–diffusion method is chosen for its consistency with the continuous space–time problem and for its optimal cost. A complete numerical analysis of this decoupling method may be found in [21,22].

The time discretization is based on a second order backward Euler differentiation formula. The diffusive terms are treated implicitly whereas the advection terms are advanced explicitly in time by a second-order extrapolation scheme. The time discretized equations can be recast in a vectorial Helmholtz system of equations for the velocity, a quasi-Poisson operator for the pressure, and a scalar Helmholtz equation for temperature to solve at each time step, noting that the box geometry allows to use fast diagonalisation method for inverting these elliptic operators at a low computational cost [23]. The time-step is constant and set equal to $\Delta t = 5 \times 10^{-3}$. More details on the DNS database can be found in [12].

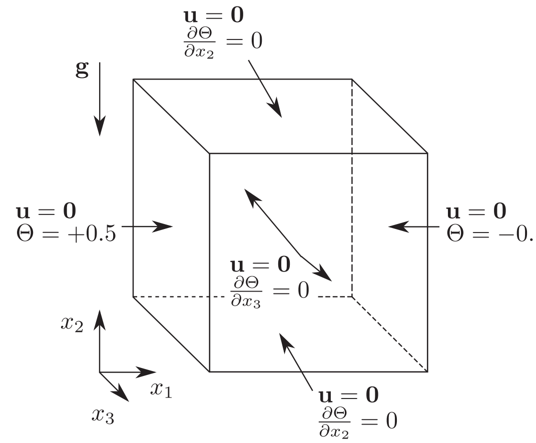

Fig. 1. Schematic view of the cubic domain and boundary conditions.

3. Proper orthogonal decomposition

This section aims to provide the mathematical grounds on which the POD method is based (for an exhaustive and detailed discussion we refer to [24–27]).

In the present context the POD is used for the identification of turbulent coherent structures of velocity, temperature and heat flux fields. It relies on a linear decomposition of a general vector field with zero mean $\phi = \phi(\mathbf{x}, t)$, $\phi \in L^2(\Omega)$, with respect to orthogonal modes (also called empirical eigenfunctions) $\zeta_j = \zeta_j(\mathbf{x})$ [15]

$$\phi(\mathbf{x}, t) \approx \phi_N(\mathbf{x}, t) = \sum_{j=1}^{N} a_j(t)\zeta_j(\mathbf{x}), \quad (9)$$

where N is the series truncation order. Given an ensemble of S realizations of the vector fields $\phi^i = \phi(\mathbf{x}, t_i)$, $\mathscr{E} = \{\phi^i\}_{i=1}^S$, we define the correlation tensor $\mathbf{R}$ of dimension $S \times S$

$$R_{jk} = \frac{1}{S}\int_\Omega \phi^j \cdot \phi^k dV = \frac{1}{S}(\phi^j, \phi^k). \quad (10)$$

It can be shown [26] that solving the eigenvalue problem

$$\mathbf{R}\mathbf{q}_j = \lambda_j\mathbf{q}_j \quad (11)$$

is equivalent to solve the maximization problem $\max\langle(\phi,\zeta)^2\rangle/(\zeta,\zeta) = \lambda$, where $\langle\cdot\rangle$ is the time-averaging operator. Finally, the empirical eigenfunctions are computed as follows:

$$\zeta_j = \sum_{i=1}^{S} q_j^i \phi^i \quad (12)$$

in such a way that $(\zeta_j, \zeta_k) = \delta_{jk}$, being δ_{jk} the Kronecker delta. Since $\mathbf{R}$ is symmetric and semi-positive defined its eigenvalues are all non-negative. We notice that the trace of the correlation tensor $\mathrm{tr}(\mathbf{R}) = \langle(\phi, \phi)\rangle = \sum_{j=1}^{S}\lambda_j$ represents the averaged *energy* content of the fluctuating vector field analysed. In natural convective flows (as well as in compressible flows or magneto hydrodynamics) two possible definitions of the state vector ϕ^i can be considered. The first

[1] The initial velocity and temperature distribution are instantaneous solutions obtained by previous computations at lower Rayleigh.