Computers & Fluids 44 (2011) 130–142

Contents lists available at ScienceDirect

Computers & Fluids

journal homepage: www.elsevier.com/locate/compfluid

Hybrid Hamilton–Jacobi–Poisson wall distance function model

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ARTICLE INFO

Article history:
Received 29 August 2010
Received in revised form 16 October 2010
Accepted 17 December 2010
Available online 8 January 2011

Keywords:
Wall distance
Differential equations
Parallel processing
Poisson equation
Hamilton–Jacobi equation
Eikonal equation
Turbulence modelling

ABSTRACT

Expensive to compute wall distances are used in key turbulence models and also for the modeling of peripheral physics. A potentially economical, robust, readily parallel processed, accuracy improving, differential equation based distance algorithm is described. It is hybrid, partly utilising an approximate Poisson equation. This also allows auxiliary front propagation direction/velocity information to be estimated, effectively giving wall normals. The Poisson normal can be used fully, in an approximate solution of the eikonal equation (the exact differential equation for wall distance). Alternatively, a weighted fraction of this Poisson front direction (effectively, front velocity, in terms of the eikonal equation input) information and that implied by the eikonal equation can be used. Either results in a hybrid Poisson–eikonal wall distance algorithm. To improve compatibility of wall distance functions with turbulence physics a Laplacian is added to the eikonal equation. This gives what is termed a Hamilton–Jacobi equation. This hybrid Poisson–Hamilton–Jacobi approach is found to be robust on poor quality grids. The robustness largely results from the elliptic background presence of the Poisson equation. This elliptic component prevents fronts propagated from solid surfaces, by the hyperbolic eikonal equation element, reflecting off zones of rapidly changing grid density. Where this reflection (due to poor grid quality) is extreme, the transition of front velocity information from the Poisson to Hamilton–Jacobi equation can be done more gradually. Consistent with turbulence modeling physics, under user control, the hybrid equation can overestimate the distance function strongly around convex surfaces and underestimate it around concave. If the former trait is not desired the current approach is amenable to zonalisation. With this, the Poisson element is automatically removed around convex geometry zones.

1. Introduction

Wall distances, d, are still a key parameter in many key turbulence models (see Baldwin and Lomax [1], Baldwin and Barth [2], Spalart and Allmaras [3], Wolfshtein [4], Secundov et al. [5], Menter [6] and Spalding [7]) and also Detached Eddy Simulations (see Shur et al. [8], Nikitin et al. [9]). They are also used for peripheral applications incorporating additional solution physics and aspects of geometric modelling (see Xia et al. [10]). For turbulence models, d is just required close to walls to a maximum of about one third of the boundary layer thickness. Surprisingly, for highly optimised RANS/URANS (Unsteady Reynolds Averaged Navier–Stokes) solvers, the effort in calculating d can be a significant fraction of the total solution time. For example, even with a Cray C90 class computer it took 3 h just to gain d (see Wigton [11]). For flows with time dependent geometry (such as Computational Aeroelasticity and design optimisation) or mesh refinement clearly this feature is exacerbated (see Boger [12]). Because of d evaluation expense in some codes dangerous approximations are made (see Spalart [13]). These can, for example, create large inaccuracies and also non-smooth, unhelpful to convergence, d distributions.

However, the careful modification of d to some $\tilde{d}$ can remedy turbulence model deficiencies or extend modelling potential. For example, in the SA [3] and ν_{t92} (Secundov et al. [5]) models roughness can be accounted for by distance modification. Also, if $\tilde{d} = d$ sharp convex features, such as a thin wire, or wing trailing edge, can have disproportionate turbulence influences (see Fares and Schroder [14]). For a thin wire in a channel the anomalous situation arises where in the wall normal direction the wire (no matter how small) has just as strong a turbulence damping influence as the channel walls. Through boosting turbulence destruction terms, wall proximity reduces eddy viscosity. Hence, the excessive influence of sharp convex features can be lessened by ensuring $\tilde{d} > d$. For corners or bodies/surfaces in close proximity the increased multiple surface turbulence damping effect (see Spalding [7], Fares and Schroder [14], Mompean et al. [15], Launder, Reece and Rodi [16]) should be taken into account. Setting $\tilde{d} < d$ is a convenient mechanism for achieving this.

Distance evaluation methods can be broadly classified as: (I) search procedures, (II) integral approaches and (III) differential equation based methods. Crude search procedures require $O(n_v n_s)$

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doi:10.1016/j.compfluid.2010.12.021

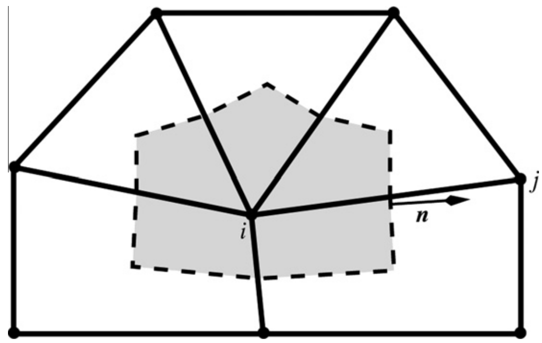

Fig. 1. Schematic of cell vertex control volume.

operations where n_s and n_v correspond to the number of surface and internal node points. Wigton [11] and Boger [12] present more efficient search procedures. These need $O(n_v\sqrt{n_s})$ and $O(n_v \log n_s)$ operations, respectively. Integral approaches are described in Boger [12], Spalart [17] and Launder et al. [16]. For complex geometries they are difficult to apply and make numerically efficient. Hence, the focus here is on a differential equation based method. Advantageously, such methods are suitable for vector and parallel computers. Some, differential equation methods are discussed below.

2. Differential equation based distance methods and new hybrid approach

2.1. Poisson equation method

Considering the above noted accuracy, or physics, considerations Spalding [7] proposed solving a Poisson differential equation for a wall distance related function φ. This equation is given below

$$\nabla^2\phi = -1 \tag{1}$$

The variable φ can be converted into a wall distance function, $\tilde{d}_P$, through the auxiliary equation below

$$\tilde{d}_p = \pm\sqrt{\sum_{j=1,3}\left(\frac{\partial\phi}{\partial x_j}\right)^2} + \sqrt{\sum_{j=1,3}\left(\frac{\partial\phi}{\partial x_j}\right)^2 + 2\phi} \tag{2}$$

The analytical derivation of (2) includes the assumption that surfaces are extensive in the non wall normal direction. Eqs. (1) and (2) have all the desirable traits, noted above i.e. for a single flat surface $\tilde{d}_P = d$, in a convex geometry region $\tilde{d}_P > d$ and in a concave region $\tilde{d}_P < d$.

2.2. Eikonal equation method

Fares and Schroder [14] also derived a differential equation for $\tilde{d}$. This again is aimed at reflecting the traits of Eqs. (1) and (2). However, an exact equation for d, that can be readily derived using coordinate geometry is the hyperbolic eikonal equation below

$$|\nabla d| = 1 + \Gamma\nabla^2 d \tag{3}$$

where $\Gamma \to 0$ giving viscosity solutions. This equation models a front propagating at unit velocity, $\tilde{\mathbf{u}}$, from surfaces. In fact, d is the first arrival time of the front. For a unit velocity this time is equivalent to wall distance. If $\Gamma = f(d)$, the front velocity is modified and the resulting Hamilton–Jacobi (HJ) equation will give the traits of Eqs. (1), (2) above (see Tucker et al. [18]. Tucker [19]) but with the potential of user control. Motivated by dimensional homogeneity and the need that as $d \to 0$, the Hamilton–Jacobi (HJ) distance function should ensure $\tilde{d}_{HJ} = d$ suggests

$$\Gamma = \tilde{\varepsilon}\tilde{d}_{HJ} \tag{4}$$

where $\tilde{\varepsilon}$ is a constant (Note, in later results $0 \leqslant \tilde{\varepsilon} \leqslant 0.5$. Finite values of $\tilde{\varepsilon}$ can improve stability, but for triangulated meshes the parameter is found of minimal benefit in this respect. However, for hexahedral cells, with rapid volume changes away from solid surfaces, as will be addressed later, the $\tilde{\varepsilon}$ level can be useful for securing iterative convergence). Here, it is proposed to hybridize Eqs. (1)–(3) creating a Poisson–Hamilton–Jacobi distance function model.

2.3. Hybrid method

If we define the pseudo velocity (which represents a front propagation velocity) shown below

$$\tilde{\mathbf{u}}_{HJ} = \nabla\tilde{d}_{HJ} \tag{5}$$

the HJ equation can be re-expressed as

$$\tilde{\mathbf{u}}_{HJ}\nabla\tilde{d}_{HJ} = 1 + \tilde{\varepsilon}\tilde{d}_{HJ}\nabla^2\tilde{d}_{HJ} \tag{6}$$

We can alternatively define a hybrid auxiliary velocity

$$\tilde{\mathbf{u}}_H = \alpha\nabla\tilde{d}_{HJ} + (1-\alpha)\frac{\nabla\tilde{d}_P}{\left|\nabla\tilde{d}_P\right|} \tag{7}$$

to compute a hybrid wall distance function $\tilde{d}_H$ For $\alpha = 1$, the HJ equation is recovered. For $\alpha < 1$ a hybrid solution that contains

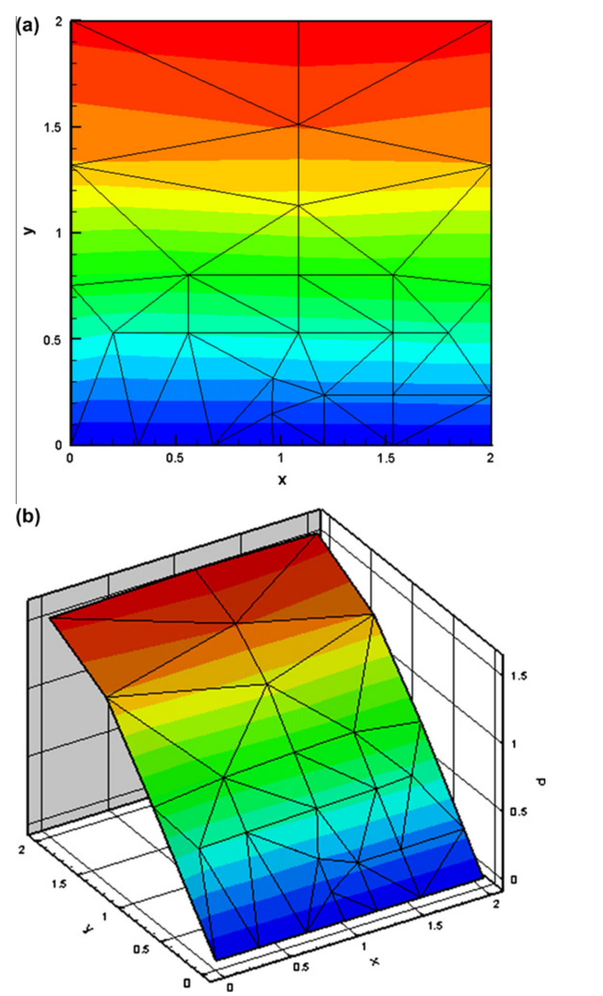

Fig. 2. HJ distance contours and grid for flat plate: (a) two-dimensional view and three-dimensional plot with vertical axis as d.