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Upwinding meshfree point collocation method for steady MHD flow with arbitrary orientation of applied magnetic field at high Hartmann numbers

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ABSTRACT

In this paper, a meshfree point collocation method, with an upwinding scheme, is presented to obtain the numerical solutions of the coupled equations in velocity and magnetic field for the fully developed magnetohydrodynamic (MHD) flow through an insulated straight duct of rectangular section. The moving least-square (MLS) approximation is employed to construct the shape functions in conjunction with the framework of the point collocation method. Computations have been carried out for different applied magnetic field orientations and a wide range of values of Hartmann number from 5 to 10^6. As the adaptive upwinding local support domain is introduced in the meshless point collocation method, numerical results show that the method may compute MHD problems not only at low and moderate values but also at high values of the Hartmann number with high accuracy and good convergence.

1. Introduction

The magnetohydrodynamic (MHD) flow is of great interest for many researchers due to its applications in broad fields, such as the cooling system with liquid metals for nuclear fission or fusion reactors, electromagnetic pumps, MHD generators. It is very important to devise effective numerical methods to obtain the approximate solutions of the MHD flow problem since the exact solutions can be achieved only in some special cases.

Several methods for solving the MHD problems numerically in two-dimensional case have been applied; however, most of them can only solve MHD problems at low or moderate Hartmann numbers. Finite element method (FEM) is one of the most popular methods for solving the MHD problems [1–4]. For FEM, most of these simulated results are fit for MHD flow at Hartmann numbers less than 100. Tezer-Sezgin applied the boundary element method to MHD flow [5] for values of Hartmann number up to 300. The analytical finite element method [6] and the element-free Galerkin method are adopted [7] to extend the range of the Hartmann numbers up to 1000. By using the residual-free bubble functions, Nesliturk and Tezer-Sezgin [8] have solved the MHD flow problems with FEM for the values of Hartmann number up to 10^5. Then Zhang et al. applied [9] element-free Galerkin method using the residual-free bubble functions and obtained the similar results for Hartmann numbers up to 10^4.

A kind of truly meshless method, meshfree point collocation method (MPCM), has been proposed to discretize directly the relevant governing equations. The meshfree point collocation method is known as its simplicity and efficiency to solve partial differential equations (PDEs) without numerical integrations. In addition, its simple formulation also attracts the attention of many researchers. Oñate et al. [10] developed a point collocation scheme for fluid flow problem on the basis of weighted least-square procedure, which they called the finite point method. The finite point method includes additional terms in the strong form to stabilize the convective term. Oñate et al. [11] also applied this method to elasticity problems. Kansa [12] solved PDEs using radial basis functions with a point collocation method for hyperbolic, parabolic, and elliptic types. Yongsik et al. [13] employed MPCM to solve the stream-vorticity formulation of two-dimensional incompressible Navier–Stokes flows.

For MHD problems, the difficulty of solving the governing equations at high Hartmann numbers is similar to that of solving the advection–diffusion equation when advection process dominates diffusion, which can be explained by the formation of layers near the walls or inside the region depending on the boundary conditions. A special treatment is needed to stabilize the numerical approximation for these kinds of problems. Upwinding schemes are one of the general techniques to stabilize FEM [14] and FVM [15]. Gu and Liu [16] employed the similar concept in the meshless methods to solve the two-dimensional convection-dominated problem with high accuracy.

A meshfree point collocation method, with an upwinding scheme (UMPCM), for two-dimensional fully developed MHD flow problems is presented in this paper. The moving least-square

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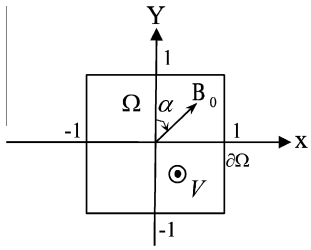

Fig. 1. Duct geometry and external applied magnetic field.

(MLS) approximation is employed to construct shape functions. A number of numerical results of MHD analyses are presented and compared with the exact [17] solutions for the flow in an insulated duct at Hartmann numbers ranged from 5 to 10^6. Furthermore, numerical solutions are also obtained for the oblique external applied magnetic field.

The organization of the rest of this paper is as follows: In Section 2 the basic equations for MHD flow are presented. In Section 3 the moving least-square approximation is outlined. In Section 4 a brief discussion of the upwinding scheme is presented. The numerical results are reported in Section 5. Finally some concluding remarks are given in Section 6.

2. The physical problem

It is well known that Maxwell equations of electromagnetism and the basic equations of fluid mechanics lead to the coupled system of equations in velocity and magnetic field. These equations are for the steady, laminar, fully developed flow of viscous, incompressible, and electrically conducting fluid in a rectangular duct Ω, subjected to a constant and uniform applied magnetic field B_0.

The external applied magnetic field B_0 lies in x–y plane of a section of the duct and forms an angle α with y-axis. z-axis is along the axial direction in which the flow is taking place for Newtonian fluid

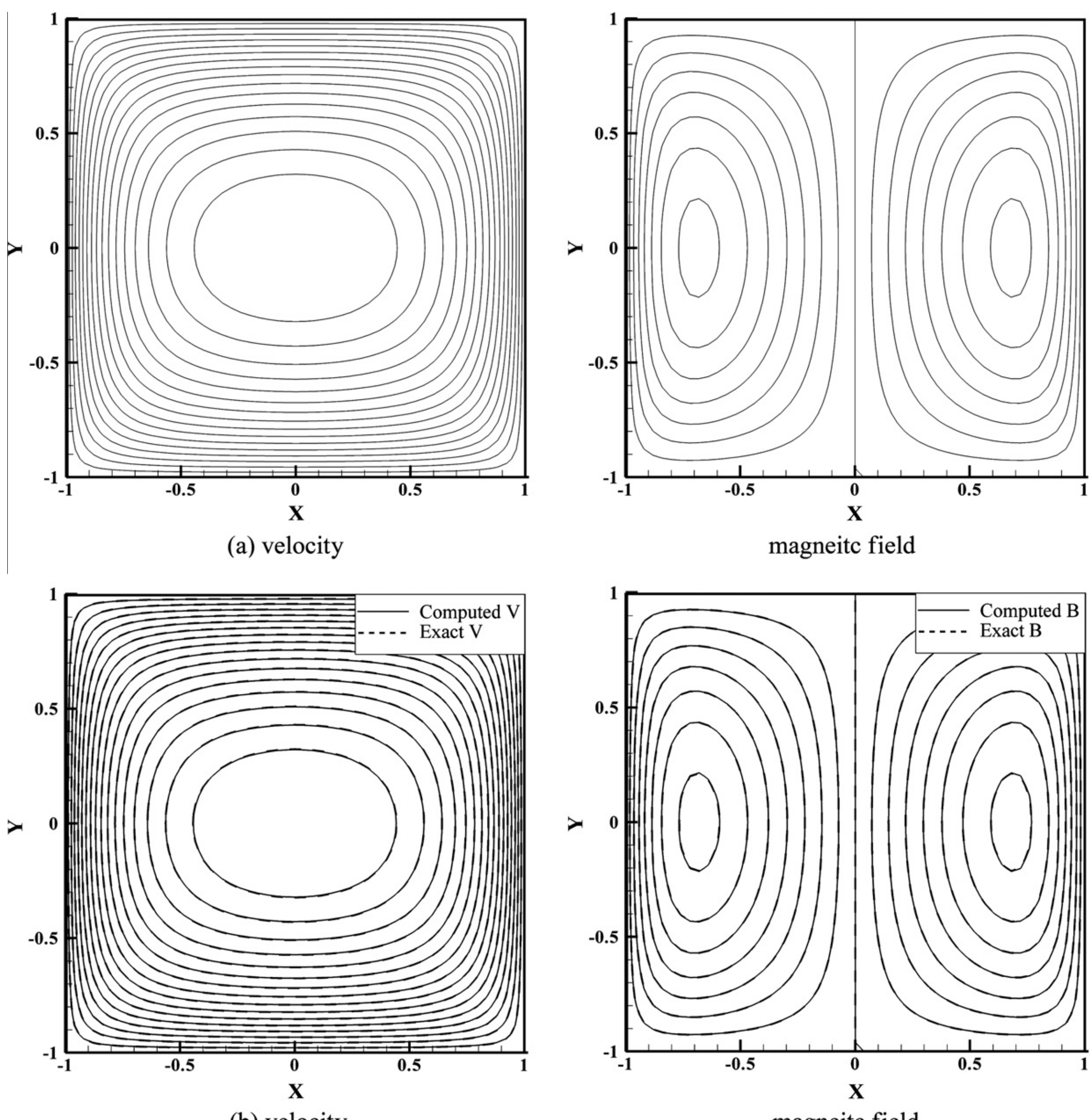

Fig. 2. Velocity and induced magnetic field for $M = 5$: (a) MPCM and (b) UMPCM.