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Numerical simulations of particle migration in a viscoelastic fluid subjected to shear flow

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ABSTRACT

Particle migration is a relevant transport mechanism whenever suspensions flow in channels with gap size comparable to particle dimensions (e.g. microfluidic devices). Several theoretical as well as experimental studies have been performed on this topic, showing that the occurring of this phenomenon and the migration direction are related to particle size, flow rate, and the nature of the suspending liquid.

In this work we perform a systematic analysis on the migration of a single particle in a sheared viscoelastic fluid through 2D finite element simulations in a Couette planar geometry. To focus on the effects of viscoelasticity alone, inertia is neglected. The suspending medium is modeled as a Giesekus fluid.

An ALE particle mover is combined with a DEVSS/SUPG formulation with a log-representation of the conformation tensor giving stable and convergent results up to high flow rates. To optimize the computational effort and reduce the remeshing and projection steps, needed as soon as the mesh becomes too distorted, a 'backprojection' of the flow fields is performed, through which the particle in fact moves along the cross-streamline direction only, and the mesh distortion is hence drastically reduced.

Our results, in agreement with recent experimental data, show a migration towards the closest walls, regardless of the fluid and geometrical parameters. The phenomenon is enhanced by the fluid elasticity, the shear thinning and strong confinements. The migration velocity trends show the existence of a master curve governing the particle dynamics in the whole channel. Three different regimes experienced by the particle are recognized, related to the particle-wall distance. The existence of a unique migration behavior and its qualitative aspects do not change by varying the fluid parameters or the particle size.

1. Introduction

Particle cross-streamline migration occurs in many practical utilizations, i.e. fluid–solid separation where different sized particles need to be separated, suspensions in microfluidic devices where migration can lead to non-homogenous particle distributions, cells in blood provoking clots obstructing the flow, removal of pollutants from a gaseous flow, etc. Recently, new applications exploiting the lateral particle motion are arising. Examples are isolation of human leukocytes [1] or focusing of red blood cells in microchannel flows for bio-sensing applications [2]. Due to its importance in applications dealing with solid-particle transport mechanisms, it has been extensively studied over many decades.

From an experimental point of view, the first work focused on particle migration was performed by Segre and Silberberg [3,4]. The authors studied the lift experienced by spheres in a dilute suspension in a Poiseuille flow at low Reynolds number. They found the existence of an equilibrium height in the channel where the particles tend to migrate.

Many analytical theories were also derived in the past, giving expressions for the lift force experienced by a particle in low Reynolds number Newtonian flows and simple geometries. An enlightening paper by Saffman [5] gave an expression for the potential lift force on a sphere in an unbounded shear flow. Such a theory was later generalized by Asmolov [6] and McLaughlin [7] removing some constraints on the flow parameters.

The motion of a sphere in the presence of a flat wall in shear flow was studied by using a perturbative approach by Leighton and Acrivos [8], Cherukat and McLaughlin [9] and Krishnan and Leighton [10]. They found the critical conditions for Reynolds number such that the sphere separates from the wall.

All the references cited above refer to a Newtonian suspending fluid. The non-Newtonian (i.e. viscoelastic) nature of the suspending medium is, however, relevant in many processes (polymer melts with fillers, rubbers, cosmetics, foods, etc.). In this case, few experimental data sets on migration are available.

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Old data by Mason and co-workers [11–13] reported on migration of spheres in viscoelastic fluids in Poiseuille and Couette flows. Only recently, a careful and complete analysis by Lormand and Phillips [14] reported on the migration of a sphere in a viscoelastic fluid in a Couette apparatus with small curvature. Their data clearly show the tendency of the particle to migrate towards the walls of the flow cell. The authors also investigated the influence of particle dimension and external flow parameters on the migration.

On the theoretical side, and limiting to simple shear only, which is the case of interest in our paper, we are aware of only two works explicitly addressing the viscoelasticity-induced cross-flow migration, but at vanishing flow rates [15] or at modest flow rates in a 2D approximation [16]. At $Re = 0$, Ho and Leal [15] predict the existence of migration induced by normal stresses whenever there is a lateral variation of the shear rate in the undisturbed flow. The first work showing migration in an inertialess viscoelastic fluid is the pioneer paper of Huang et al. [16] where Direct Numerical Simulations are performed. They reported a particle lateral motion with direction depending on the presence/absence of a rate-dependent viscosity of the suspending liquid. In their numerical simulations, the influence on particle lift of fluid parameters, particle size/channel height ratio were also studied. Huang et al. [16] accounted for shear thinning with an ad-hoc modification of the Oldroyd-B model. They used an elastic–viscous-split-stress (EVSS) formulation to discretize the momentum balance and the solid–fluid coupling is treated by an Arbitrarian Lagrangian Eulerian (ALE) method with mesh velocities. As in Ho and Leal [15], the main conclusion of Huang et al. [16] is that migration has to be ascribed to the presence of normal stresses.

The purpose of the present work is to perform a systematic numerical study on the migration of a particle suspended in a realistic viscoelastic fluid under simple shear flow. The suspending medium is modeled as a Giesekus fluid [17], which is often capable of accurately describing experimental viscoelastic data. The study is carried out by neglecting fluid and particle inertia. It is anticipated here that our simulation results qualitative agree with the experiments by Lormand and Phillips [14], i.e., migration is predicted to occur towards the walls of the shear cell, although our results are limited to 2D. The analysis is performed through 2D direct numerical simulations to take into account the non-linear nature of the problem.

The momentum balance is discretized through the Discrete-Elastic–Viscous-Split-Stress (DEVSS) method that is one of the most robust formulations currently available. The viscoelastic constitutive equation is stabilized by implementing the Streamline-Upwind-Petrov–Galerkin (SUPG) technique. Furthermore a log-conformation representation of the conformation tensor is used. Finally, an ALE particle mover [18] is adopted to handle the particle motion. The numerical scheme implemented here leads to stable and convergent results, and allows one to achieve substantial flow rates (as compared with [16]). In order to efficiently manage the particle motion, a trick is used whereby the particle only moves along the y-direction (i.e., the migration direction). This is achieved through 'backprojection' of the computed fields along the x-direction (the main flow direction) at any time step. In this way, remeshing due to ALE approach is only needed once–twice per run, always preserving the accuracy of the solution.

The influence of flow rate as well as of particle dimension (as compared to the gap size) on the migration velocity is also studied.

2. Governing equations

We consider a single, rigid, non-Brownian, inertialess, circular particle (2D problem) moving in a channel filled by a viscoelastic fluid. The problem is schematized in Fig. 1: a particle with diameter $D_p = 2R_p$, denoted by $P(t)$ and boundary $\partial P(t)$, moves in a rectangular domain, Ω, with dimensions L and H along x- and y-axis respectively and external boundaries denoted by Γ_i $(i = 1, \ldots, 4)$. The Cartesian x and y coordinates are selected with the origin at the center of the domain. On the upper and lower boundaries, equal and opposite velocities are imposed that, for an unfilled fluid, would generate the shear flow depicted on the right part of the same figure.

The vector $\boldsymbol{x}_p = (x_p, y_p)$ gives the position of the center of the particle P. In order to evaluate particle rotation, an angular information, $\boldsymbol{\Theta} = \Theta \boldsymbol{k}$, is also associated with the particle, where $\boldsymbol{k}$ is the unit vector in the direction normal to the x—y plane. The particle moves according to the imposed flow and its rigid-body motion is completely defined by the translational velocity, denoted by $\boldsymbol{U}_p = d\boldsymbol{x}_p/dt = (U_p, V_p)$ and angular velocity, $\boldsymbol{\omega} = d\boldsymbol{\Theta}/dt = \omega \boldsymbol{k}$.

The governing equations for the fluid domain, $\Omega - P(t)$, neglecting inertia, can be stated as follows:

$$\nabla \cdot \boldsymbol{\sigma} = \boldsymbol{0} \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0 \tag{2}$$

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\eta_s \boldsymbol{D} + \boldsymbol{\tau} \tag{3}$$

Eqs. (1)–(3) are the equations for the momentum balance, the mass balance (continuity) and the expression for the total stress, respectively. In these equations $\boldsymbol{u}$, $\boldsymbol{\sigma}$, p, $\boldsymbol{I}$, $\boldsymbol{D}$, η_s, are the velocity vector, the stress tensor, the pressure, the 2×2 unity tensor, the rate-of-deformation tensor and the viscosity of a Newtonian 'solvent', respectively. The viscoelastic stress, $\boldsymbol{\tau}$, is written as (for the constitutive model chosen, see below):

$$\boldsymbol{\tau} = \frac{\eta}{\lambda}(\mathbf{c} - \boldsymbol{I}) \tag{4}$$

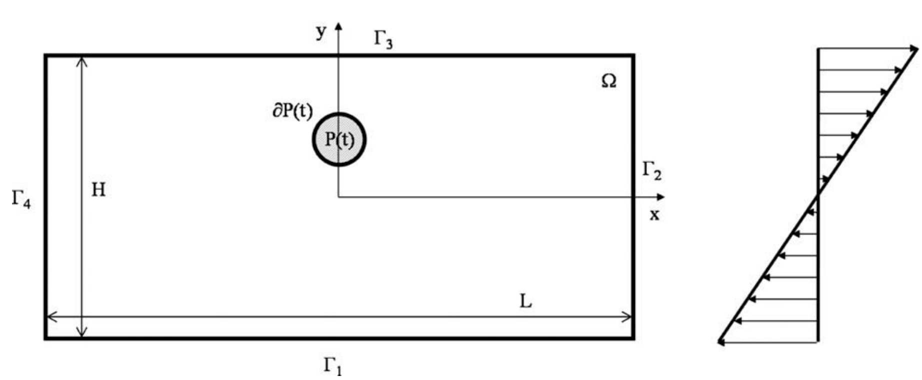

Fig. 1. Schematic representation of the problem: a rectangular fluid domain (Ω) with dimensions $L \times H$ with a single particle $P(t)$ is considered. The origin of a Cartesian frame is located at the domain center.