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MHD flow in porous medium induced by torsionally oscillating disk

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ABSTRACT

The unsteady laminar flow of an incompressible, viscous, electrically conducting fluid in porous medium fully saturated with the liquid and bounded by torsionally oscillating disk in the presence of a transverse magnetic field has been computed. It is assumed that the flow between the disk and the porous medium is governed by Navier–Stokes equation and that in the porous medium by Brinkman equation. Flows in the two regions are matched at the interface by assuming that the velocity and stress components are continuous at it. Approximate solutions of the flow characteristics are obtained. Numerical results are presented graphically and discussed.

1. Introduction

The flow of a viscous liquid over and through porous medium has been the subject of intensive studies in recent year because of its natural occurrence in the movement of water and oil inside the earth and the flow of river through porous banks. It has applications in many engineering and biomedical problems. For example, the flow of liquid in porous bearing and porous rollers was studied by Joseph and Tao [1]. The mathematical theory of the flow of liquid through a porous medium was initiated by Darcy [2]. For the steady flow he assumed that viscous forces were in equilibrium with external forces due to pressure difference and body forces. Brinkman [3] proposed modification of the Darcy's law for porous medium, which was assumed to be a swarm of homogeneous spherical particles. Srivastava and Sharma [4] studied the flow and heat transfer of a viscous liquid confined between a rotating plate and a porous medium by assuming that the flow in the porous medium was governed by Brinkman equation [3] and that in the free flow region by Navier–Stokes equation. Rosenblat [5] examined the flows resulting from the small torsional oscillations of an infinite disk in a viscous fluid otherwise unbounded and at rest. Rosenblat [6] also studied the case when two parallel plane disks, between which a viscous liquid was confined, oscillated torsionally about a common axis. Srivastava [7] studied the torsional oscillation of an infinite disk in a second order fluid when fluid was unbounded as well as bounded by a parallel impervious disk. Also, Srivastava [8] studied torsional oscillation of a disk near its surface in presence of porous medium.

In recent years, the requirements of modern technology have stimulated interest in fluid flow studies, which involve the interaction of several phenomena. The subject of hydromagnetics has attracted the attention of many authors, due not only to its own interest, but also to many applications to the problems of geophysical and astrophysical significance. In view of its wide applications in industrial and other technological fields the problem of flow near a rotating disk has been extended to hydromagnetics first investigated by Katukani [9], and Sparrow and Cess [10]. Hughes and Elco [11] discussed the magnetohydrodynamics lubrication flow between parallel rotating disks. The magnetohydrodynamics flow between eccentrically rotating disks have also been studied by Ramachandra Rao and Raghupathi Rao [12]. Also, the numerical solution of the MHD flow near a rotating disk has been given by Watanabe and Oyama [13], and Kumar et al. [14]. Recently, Ariel [15] studied the MHD flow near a rotating disk.

The object of the present paper is to study the flow of viscous incompressible, electrically conducting fluid in porous medium fully saturated with the liquid and bounded by torsionally oscillating disk in the presence of magnetic field. It is assumed that the free flow region of clear fluid is governed by Navier–Stokes equation and the flow in the porous medium is governed by Brinkman equation [3].

2. Statement of the problem

Consider the flow of an incompressible viscous fluid confined between an impervious disk performing torsional oscillation of frequency n, angular speed Ω and a porous medium fully saturated with the fluid. Let (r^*, θ, z^*) be a set of cylindrical polar coordinates and let the oscillating disk be represented by the plane $z^* = d$, the

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interface by $z^* = 0$ and the porous region $z^* < 0$. A magnetic field of constant intensity $\vec{B}$ is applied perpendicular to the disk. In order to derive the basic equation for the region I between $z^* = 0$ and $z^* = d$ the following assumption are made

(1) The flow is unsteady and the fluid is viscous, incompressible and finitely conducting with constant physical properties.
(2) The magnetic Reynold number is taken to be small enough so that the induced magnetic field is neglected.
(3) Hall effect, electrical and polarization effects are neglected.

Under these assumptions, we write equations governed by the following Navier–Stokes equation and equation of continuity as

$$\rho\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right] = -\nabla p + \mu\nabla^2\vec{v} + (\vec{J}\times\vec{B}) \quad (1)$$

where the third term on the right hand side of Eq. (1) is the Lorentz force due to magnetic field $\vec{B}$, and is given by

$$\vec{J}\times\vec{B} = \sigma(\vec{v}\times\vec{B})\times\vec{B}$$
$$\nabla\cdot\vec{v} = 0 \quad (2)$$

respectively, where ρ, μ, p are respectively density, viscosity, pressure of the fluid and $\vec{v}$ is the velocity vector at any point. The porous region $z^* < 0$ is called region II and in this region the flow is governed by Brinkman equation [3] and the equation of continuity given by

$$\rho\frac{\partial \vec{v}}{\partial t} = -\nabla P + \mu_e\nabla^2\vec{v} - \frac{\mu\vec{v}}{k} + (\vec{J}\times\vec{B}) \quad (3)$$
$$\nabla\cdot\vec{v} = 0 \quad (4)$$

respectively, where k is the permeability of the porous medium and $\vec{v}$, P are the velocity vector and pressure at any point in the porous medium.

The coefficient μ_e is the effective viscosity for the Brinkman flow model, which is different from μ, the viscosity of the fluid. Givler and Altobelli [16] have determined experimentally μ_e for steady flow through a wall-bounded porous medium and their results show that $\mu_e = (7.5^{+3.4}_{-2.4})\mu$.

Let u, v, w be the velocity components in the direction of r^*, θ, z^* respectively in the region I and corresponding components in the region II are U, V, W. Then the boundary conditions of the problem are

$$u = 0,\ v = r^*\Omega\cos nt,\ w = 0 \quad \text{on } z^* = d \quad (5)$$
$$U \to 0,\ V \to 0 \quad \text{as } z^* \to -\infty \quad (6)$$

The boundary condition at the interface of the porous medium and clear fluid $z^* = 0$, we assume the velocity components and the pressure are continuous and the jumps in shearing stresses $\tau_{z\,\theta}$ and $\tau_{z\,r}$ are given the equation suggested by Ochao-Tapia and Whittaker [17,18]. These assumptions in our notation can be written as

$$u = U,\ v = V,\ w = W,\ p = P \quad \text{at } z^* = 0 \quad (7)$$
$$\mu_e\frac{\partial U}{\partial z^*} - \mu\frac{\partial u}{\partial z^*} = \beta\frac{\mu}{\sqrt{k}}U \quad \text{at } z^* = 0 \quad (8)$$
$$\mu_e\frac{\partial V}{\partial z^*} - \mu\frac{\partial v}{\partial z^*} = \beta\frac{\mu}{\sqrt{k}}V \quad \text{at } z^* = 0 \quad (9)$$

3. Equations of motion

We assume the following form of velocity components for the region I

$$u = rd(\Omega^2/n)\frac{\partial f(z,\tau)}{\partial z}, \quad v = rd\Omega e^{i\tau}g(z),$$
$$w = -2d(\Omega^2/n)f(z,\tau), \quad p^* = \rho\Omega^2 d^2 p(z,\tau) \quad (10)$$

$$z = z^*/d, \quad r = r^*/d, \quad \tau = nt \quad (11)$$

Here the complex notation is adopted with the convention that only real parts of the complex quantities represent physical quantities. Writing Eq. (1) in cylindrical polar coordinates and substituting (10) we get the following equations of motion in the direction of r and θ respectively

$$\frac{\partial^2 f}{\partial z\partial t} + \left(\frac{\Omega}{n}\right)^2\left[\left(\frac{\partial f}{\partial z}\right)^2 - 2f\frac{\partial^2 f}{\partial z^2}\right] - (ge^{i\tau})^2 = \frac{1}{\text{Re}}\frac{\partial^3 f}{\partial z^3} - \frac{M^2}{\text{Re}}\frac{\partial f}{\partial z} \quad (12)$$

$$ig + 2\left(\frac{\Omega}{n}\right)^2\left[g\frac{\partial f}{\partial z} - f\frac{\partial g}{\partial z}\right] = \frac{1}{\text{Re}}\frac{\partial^2 g}{\partial z^2} - \frac{M^2}{\text{Re}}g \quad (13)$$

where $\text{Re} = \rho n\ d^2/\mu$ is the Reynold's number of the flow and $M = \sqrt{(\sigma B^2 d^2/\mu)}$ is the Hartmann number. The equation in the direction of z^* serves merely to determine the axial pressure gradient and hence is not given. Assume the following form of the velocity components for the region II

$$U = rd(\Omega^2/n)\frac{\partial F(z,\tau)}{\partial z}, \quad V = rd\Omega e^{i\tau}G(z),$$
$$W = -2d(\Omega^2/n)F(z,\tau), \quad P^* = \rho\Omega^2 d^2 P(z,\tau) \quad (14)$$

Again writing (3) in cylindrical polar coordinates and substituting (14) we get the following equation in the direction r and θ respectively

$$\frac{\partial^2 F}{\partial z\partial t} - (Ge^{i\tau})^2 = \frac{\gamma^2}{Re}\frac{\partial^3 F}{\partial z^3} - \frac{M^2}{\text{Re}}\frac{\partial F}{\partial z} - \frac{\sigma^2}{\text{Re}}\frac{\partial F}{\partial z} \quad (15)$$

$$iG = \frac{\gamma^2}{\text{Re}}\frac{\partial^2 G}{\partial z^2} - \frac{M^2}{\text{Re}}G - \frac{\sigma^2}{\text{Re}}G \quad (16)$$

where $\sigma = d/\sqrt{k}$ is the Darcy number and $\gamma^2 = \mu_e/\mu$. The boundary conditions (5) and (6) can be written as

$$f = \frac{\partial f}{\partial z} = 0, \quad g = 1 \quad \text{at } z = 1 \quad (17)$$
$$\frac{\partial F}{\partial z} \to 0, \quad G \to 0 \quad \text{as } z \to -\infty \quad (18)$$

Conditions (7)–(9) at the interface can be written as

$$f = F, \quad \frac{\partial f}{\partial z} = \frac{\partial F}{\partial z}, \quad \gamma^2\frac{\partial^2 F}{\partial z^2} - \frac{\partial^2 f}{\partial z^2} = \beta\sigma\frac{\partial F}{\partial z} \quad \text{at } z = 0 \quad (19)$$
$$g = G, \quad \gamma^2\frac{dG}{dz} - \frac{dg}{dz} = \beta\sigma G \quad \text{at } z = 0 \quad (20)$$

4. Transverse component of the velocity

Assuming the amplitude of the oscillation $\left(\frac{\Omega}{n}\right)$ to be small we can neglect the non-linear convective term in Eq. (13). The equation for transverse components of the velocity in the region I is given by

$$\frac{d^2 g}{dz^2} - (i\text{Re} + M^2)g = 0 \quad (21)$$

The magnitude of neglecting terms and the justification for neglecting them will be given in the last section. The transverse component of the velocity for the region II is governed by the Eq. (16). The solution of Eqs. (16) and (21) satisfied the boundary conditions (17) and (18) together with condition (20) at the interface is given by

$$g(z) = \frac{(c_1 + ic_2)e^{-(a+ib)z} - (c_3 + ic_4)e^{(a+ib)z}}{(c_7 + ic_8)} \quad (22)$$

$$G(z) = \frac{(c_5 + ic_6)e^{(A+iB)z}}{(c_7 + ic_8)} \quad (23)$$