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ORIGINAL ARTICLE

Interpolation of parametric CNC machining path under confined jounce

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Abstract Aiming at reducing the computer numerically controlled (CNC) machining vibration and increasing machining quality, an interpolation method for parametric tool paths with confined jounce, jerk, acceleration, and speed is proposed. An acceleration/deceleration profile with confined jounce, jerk, acceleration, and speed is proposed, and it is shown that this profile is time optimal to change the speed from one value to another under the given constraints. For a given parametric tool path, the velocity function is obtained by first computing the critical points of the tool path where the radius of curvature reaches extremal values, then determining the feasible maximal speeds at the critical points, and finally using the jounce confined acceleration/deceleration profile to connect the speeds at the adjacent critical points. A vibration experiment is conducted, which shows that vibration of the CNC machine decreases significantly for motions under confined jounce than that under confined acceleration and jerk. Simulation for two real CNC models are given to show the feasibility of the method.

Keywords Interpolation · Velocity planning · Confined jounce · Time optimal · Parametric curve · Vibration · High-precision CNC machining

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1 Introduction

Interpolation algorithms, which control how the computer numerically controlled (CNC) machine tool moves along the machining path, is one of the important factors in high speed and high precision CNC machining. In particular, proper kinematic controls and interpolation algorithms are essential to reduce vibration and achieve high-precision CNC machining [12]. An interpolation algorithm in the CNC controller usually consists of two phases: velocity planning and parameter computation. Let $C(u)\, u \in [0, 1]$ be the machining path. The phase to determine the velocity function $v(u)$ along $C(u)$ is called velocity planning. When the velocity function $v(u)$ is known, the phase of sampling or computing the next interpolation point at parametric value $u_{i+1} = u_i + \triangle u$ during one sampling period is called parameter computation.

A key factor to compute the velocity function during velocity planning is to choose an acceleration/deceleration (AD) profile. An AD profile is the procedure to use different acceleration and deceleration modes to change the speed from one value to another under certain constraints.

The simplest AD profile is the linear AD profile, where the acceleration and speed are bounded or confined. Using a linear AD profile for each axis, Borow [2], Shiller et al. [13], and Farouki and Timar [14] presented a time-optimal velocity planning method for a parametric path. Zhang et al. simplified the method in [14] for quadratic B-splines and realized real-time machining on an industrial CNC machine [21]. Yuan, Zhang et al. presented a time-optimal velocity planning method with confined acceleration and chord error [18, 20].

In a linear AD profile, the acceleration can change instantaneously from its maximal value A to $-A$, resulting in large vibration of the CNC machine and decreasing the machining quality. A better AD profile is the *S-shape* profile, where the jerk is confined. The S-shape AD profile is widely used in velocity planning [3, 6, 8–11, 15, 19] to generate motion with confined jerk. Another method to obtain velocity functions with confined jerk is to use trigonometric function profiles [7]. Comparing to the S-shape profile, the acceleration function obtained with trigonometric function profiles is not differentiable. It also does not have the time-optimal property of the S-shape profile similar to Theorem 2.5 in this paper.

The S-shape AD profile increases the smoothness of the velocity function. However, the jerk can jump from its maximal value to its minimal value. Since change of the jerk reflects the non-smooth change of acceleration, the profile could still result in vibration and decrease machining quality. Gai et al. [5] used an average filter to improve the S-shape AD profile to obtain a continuous jerk in some extent. But this method fails to guarantee the continuity of acceleration and is not time optimal.

A natural way to obtain smoother velocity functions is to use confined jounce. Let F be the driving force of the CNC servo and F_c the combined cutting force and friction force. Denote the mass of the axes to be m and let j be jerk. Differentiate j to obtain

$$\frac{d^2(F - F_c)}{d^2 t} = m\frac{dj}{dt} = ms, \tag{1}$$

where s is called the *jounce* or *snap*, which reflects the instantaneous change of jerk. If the jounce is confined, a motion with continuous jerk, differentiable acceleration, and C^2 velocity will be obtained. The main contribution of this paper is to design and investigate an AD profile with confined jounce and to present an interpolation algorithm for parametric tool paths based on the jounce confined AD profile.

The key component of the AD profile is a seven-period acceleration profile to increase the speed from one value to another. The whole AD profile first uses a seven-period acceleration profile to increase the speed from the initial value to its maximal value, then stays at the maximal value for a while, and finally uses a seven-period deceleration profile to decrease the speed from its maximal value to the end value. This paper analyzes the properties of the AD profile in detail and prove that this profile is time optimal to change the speed from one value to another under the given constraints.

For a curved tool path given by a set of parametric equations, the sensitive corner approach similar to that used in [7, 17] is used to compute its velocity function. Firstly, the critical points of the tool path, where the radius of curvature reaches extremal values, are computed. From these extremal values and a given sampling period, the maximal speeds that can be reached in these critical points are determined. Secondly, a backtracking procedure is used to check the reachability of the maximal speeds at each pair of neighboring critical points, that is, whether it is possible to change the maximal speed at one critical point to the maximal speed at the next critical point within the given tool path length. In the unreachable case, the speeds at the critical points are adjusted to make them reachable. Finally, between each pair of critical points, use the jounce confined AD profile to bridge their speeds, and the final velocity function is the combination of these AD profiles.

A vibration test experiment is carried out to compare AD profiles with confined acceleration, jerk, and jounce, which shows that the AD profile with confined jounce indeed can be used to reduce the vibration of the CNC machine tools and hence is useful to improve CNC machining quality. Furthermore, simulation results are given for two CNC models to show the feasibility of the proposed method.

The rest of this paper is organized as follows: In Section 2, the seven-period AD profile with confined jounce is designed and analyzed. In Section 3, the interpolation algorithm for parametric tool paths is proposed. In Section 4, experimental and simulation results are given. In Section 5, the paper is concluded.

2 An AD profile with confined jounce, jerk, acceleration, and speed

In this section, an AD profile for the speed to change from a start value v_s to an end value v_e within a distance d_m and under confined jounce will be presented. The profile is time optimal under the given conditions. Since the obtained velocity function has confined jounce, it is C^2 continuous.

2.1 A seven-period acceleration profile

In this section, a seven-period acceleration profile to increase the speed from zero to a given value $v_m > 0$ is introduced. The profile will serve as a basic step of our AD profile to be presented in Section 2.4.

The seven-period acceleration profile is given in Fig. 1, where the horizontal axis is time t. The profile is used to increase the speed from zero to $v_m > 0$ in seven periods as marked in the figure. If the speed is