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A method to couple HEM and HRM two-phase flow models

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ABSTRACT

We present a method for the unsteady coupling of two distinct two-phase flow models (namely the Homogeneous Relaxation Model, and the Homogeneous Equilibrium Model) through a thin interface. The basic approach relies on recent works devoted to the interfacial coupling of CFD models, and thus requires to introduce an interface model. Many numerical test cases enable to investigate the stability of the coupling method.

1. Introduction

We focus in this paper on the unsteady interfacial coupling of two distinct two-phase flow models that are commonly used in order to simulate water–vapor flows in nuclear power plants. We emphasize that we only deal with a *steady* coupling interface that separates the two codes. The main objective here is to prescribe meaningful boundary conditions on each side of this coupling interface for both codes associated with HEM and HRM models.

The Homogeneous Relaxation Model (denoted by the acronym HRM) is a four-equation model that is widely used in two-phase flow simulations. Most industrial codes within the nuclear community – for instance THYC (EDF) or FLICA (CEA) – rely on this model. This model requires computing approximations of solutions of two mass balance equations, a total momentum equation and a governing equation for the total energy balance of the mixture. Excluding source terms, this model is under conservative form. The only non-zero source contribution is on the right hand side of the governing equation of the liquid mass fraction. This source tends to relax the current liquid mass fraction to the equilibrium mass fraction, which only depends on the mean pressure and the mean density. The underlying time scale is highly variable, and in practice it makes the source term very stiff, which may render the computation of the HRM model rather uneasy. Actually, the Homogeneous Equilibrium Model (acronym HEM) precisely stands for the counterpart of the HRM model when *an equilibrium is achieved*. It is thus a pure convective set of partial differential equations which govern the motion of the total mass, the global momentum and the total energy of the whole mixture. Both the HEM and the HRM models require defining appropriate equations of state (referred to as the EOS in the following) in order to account for both the "pure vapor" phase, the "pure liquid" phase but also the "mixture" phase. These EOS are usually tabulated (see [30,31,22]), but we will focus here on simplified analytical EOS. This is essentially motivated by the fact that we do not wish to mix numerical drawbacks due to the use of realistic EOS and those connected with the formulation of the coupling techniques. In other words, we want to be "optimal" in some sense in terms of EOS in order to concentrate on the main drawbacks of the coupling techniques.

In order to introduce the problem of the interfacial coupling of two existing codes, we need to define governing equations:

$$\partial_t(W) + \partial_n(F_{n,L}(W)) = 0 \tag{1}$$

For the left code ($x_n = x \cdot n < 0, t > 0$), respectively, for the right code ($x_n > 0, t > 0$):

$$\partial_t(W) + \partial_n(F_{n,R}(W)) = 0, \tag{2}$$

where n is the unit normal to the plane and steady coupling interface, which is located at $x_n = 0$. Moreover, we assume that the two systems on each side are hyperbolic and invariant under frame rotation.

Quite recently, some authors have proposed two approaches in order to tackle the unsteady interfacial coupling of CFD models. Roughly speaking, the first approach favors the continuity of the conservative variable W, by enforcing $W(x_n = 0^-, t) = W(x_n = 0^+, t)$ in a weak sense (see [18,20]). This method has been recently extended to the case of a generic variable $Z(W)$ (see [4,2] and also [13,10,3]). The second one relies on the basic paper by Greenberg and Leroux [21]. It consists in introducing a color function $Y(x,t)$ where $Y(x,t) = 1$, if $x_n = x \cdot n < 0$, and $Y(x,t) = 0$ if $x_n = x \cdot n > 0$. Since the interface is steady, the function Y verifies $\partial_t(Y) = 0$. Defining $F_n(W) = YF_{n,L}(W) + (1-Y)F_{n,R}(W)$, the fluxes at the steady coupling interface can be computed by solving the Riemann problem associated with:

$$\partial_t(W) + \partial_n(F_n(W)) = 0, \tag{3}$$

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This method, which introduces the "father model" (3) obviously privileges the conservation law. It has been used in [23,26].

More recently, a third approach has been proposed [4,2]. It combines the second method with the relaxation methods [11,5,6]. The coupling technique that is used herein makes use of the latter approach. Actually, we want here to take advantage of the fact that the HRM model may be viewed as the "father model" of the HEM model. Another advantage of the third approach is that one may get rid of possible resonance phenomena, as underlined in [2,4] for instance. This phenomena may arise when using the second approach if a genuinely non linear field overlaps the steady linearly degenerate field associated with the color function Y. Though it is not clear whether this has drastic consequences, it seems indeed much more reasonable to avoid this problem that is not clearly understood [17].

The paper is organized as follows:

- Sections 2 and 3 are devoted to the presentation of both HEM and HRM models, but also on some of their properties (hyperbolicity, entropy inequality, positivity results for sufficiently smooth solutions).
- We then present the coupling method in Section 4. Special attention will be paid to the numerical treatment of the coupling interface, which relies on:
  (i) an evolution step,
  (ii) an instantaneous relaxation step,
  (iii) a finite relaxation step in order to account for source terms.

This section also includes a brief description of the Finite Volume methods that will be applied in order to compute approximations of solutions in non coupled codes.

- Numerical results are displayed in Section 5. This includes basic test cases involving contact waves, shock waves and rarefaction waves, but also a schematic representation of the flow in a part of the primary coolant circuit in a nuclear power plant.

Throughout the paper, we will use the following notations: ρ stands for the density of the mixture, $\tau = 1/\rho$ is its specific volume and U represents the mean velocity of the mixture. Moreover P, C, e, $h = e + P/\rho$, $E = e + U^2/2$, respectively, stand for the pressure, the liquid mass fraction, the internal energy, the enthalpy, and the total volumetric energy of the mixture. The subscripts v and l, respectively, refer to the *vapor* and the *liquid* phases. The overscript s denotes saturated quantities.

2. The homogeneous relaxation model

This four-equation model can be derived from the six-equation two-fluid model. In the following we focus on specific closure laws and we detail some properties connected with these choices.

2.1. Closure laws

We consider that the two fluids have the same mean velocity, that is $U_l - U_v = U_r = 0$. In order to take into account the mass transfer between the two phases, a source term $\rho\Gamma$ stands on the right hand side in the equation of the mass balance of the liquid phase. Thus the governing equations are:

$$\begin{cases} \partial_t(\rho C) + \partial_x(\rho C U) = \rho\Gamma \\ \partial_t(\rho) + \partial_x(\rho U) = 0 \\ \partial_t(\rho U) + \partial_x(\rho U^2 + P) = 0 \\ \partial_t(\rho E_{\mathrm{HRM}}) + \partial_x(U(\rho E_{\mathrm{HRM}} + P)) = 0 \end{cases} \tag{4}$$

with:

$$E_{\mathrm{HRM}} \overset{def}{=} e_{\mathrm{HRM}}(P,\rho,C) + \frac{U^2}{2} \quad \text{and} \quad e_{\mathrm{HRM}}(P,\rho,C) \overset{def}{=} h_{\mathrm{HRM}}(P,\rho,C) - \frac{P}{\rho} \tag{5}$$

where the function $h_{\mathrm{HRM}}(P,\rho,C)$ is the specific enthalpy that must be prescribed by the user. In practice here, we will use the definition (10).

The set of physical relevant states for the system (4) is:

$$\Omega_{\mathrm{HRM}} \overset{def}{=} \{(\rho, U, P, C)/\rho \geqslant 0, C \in [0,1], P \geqslant 0\} \tag{6}$$

In order to close the system, we need to define h_{HRM} and Γ.

- First, we write the enthalpy function h_{HRM}. We choose the thermodynamic closures inspired by the THYC and FLICA codes [31,22]. They consider the medium as the mixing of two fluids: liquid water and vapor. Moreover, the vapor is assumed to be in a saturation state, this implies that each thermodynamic function relative to this fluid only depends on one variable, say the pressure P. The two pure fluids are assumed to obey a usual γ closure law.

  We note $\gamma_v > 1$ (respectively, $\gamma_l > 1$) the adiabatic constant for the vapor (respectively, the liquid), and e_v, h_v, ρ_v and τ_v the internal specific energy, specific enthalpy, density and specific volume for the vapor (respectively, e_l, h_l, ρ_l and τ_l the internal specific energy, specific enthalpy, density and specific volume for the liquid).

  We recall the following definitions:

$$e_p(\rho,P) \overset{def}{=} \frac{P}{(\gamma_p - 1)\rho} \quad \text{and} \quad h_p(\rho,P) \overset{def}{=} \delta_p P \tau \quad \text{with} \quad \delta_p \overset{def}{=} \gamma_p/(\gamma_p - 1) \tag{7}$$

for $p = l, v$.We will use standard values in order to account for liquid and gas, respectively: $\gamma_l = 1.001$ and $\gamma_v = 1.4$. Moreover, we assume that if the pressure P stays in $[P_{MIN}, P_{MAX}]$ the saturation curves for the enthalpy and the volumetric fraction can be approached by the following functions:

– Saturated vapor:

$$h_v^s(P) \overset{def}{=} A_v P + B_v \quad \text{and} \quad \tau_v^s(P) \overset{def}{=} \frac{h_v^s(P)}{\delta_v P} \tag{8}$$

– Saturated liquid

$$h_l^s(P) \overset{def}{=} A_l P + B_l \quad \text{and} \quad \tau_l^s(P) \overset{def}{=} \frac{h_l^s(P)}{\delta_l P} \tag{9}$$

Physically relevant saturation curves ensure that: $\tau_v^s(P) > \tau_l^s(P)$ and $h_v^s(P) > h_l^s(P)$. Typical values of the coefficients P_{MIN}, P_{MAX}, $A_v < 0$, A_l, B_v and B_l related to nuclear cooling conditions can be found at the beginning of Section 5.

We can now define the total specific enthalpy of the mixture:

$$h_{\mathrm{HRM}}(P,\rho,C) \overset{def}{=} C h_l(P,\rho_l) + (1-C) h_v^s(P), \tag{10}$$

where

$$\rho_l = \rho C \frac{1}{1 - \rho(1-C)\tau_v^s(P)} = \frac{C}{\tau - (1-C)\tau_v^s(P)}, \tag{11}$$

This relation (11) is obtained by introducing the two void fractions α_l and α_v for the liquid and vapor, respectively, that are in agreement with $\alpha_l + \alpha_v = 1$. Using the standard definitions of the partial mass for the liquid and the mean density of the mixture:

$$\alpha_l \rho_l = \rho C \quad \text{and} \quad : \rho = \alpha_l \rho_l + \alpha_v \rho_v$$

one may eliminate $\alpha_{l,v}$ and inject $\rho_v = \rho_v^s(P)$, in order to obtain ρ_l in terms of P, ρ, C, that is the above relation (11).

In the following, the HRM model will refer to the set of Eqs. (4, 5, 10).

The enthalpy (10) of the model can be simplified, by using (7, 8).