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One-information suboptimal control repercussion on the fine structure of wall turbulence

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Sedat F. Tardu dedicates this paper to the memory of Esat Tardu, his father.

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ABSTRACT

The suboptimal control with the cost function directly connected to the wall shear and introduced for a while has been revisited through direct numerical simulations of high temporal and spatial resolution. Its effect on the fine structure of the wall turbulence has been analyzed in details, essentially through the spanwise vorticity transport mechanism. It is shown that only half of the viscous sublayer is mainly affected by the control. The actuation efficiency is limited in terms of the wall shear stress reduction, but is high as long as the turbulent wall activity is concerned. The wall shear stress is reduced due both to the reduction of the shear production in the viscous sublayer and to the contribution of the turbulent body force. The dissipation involving in the streamwise vorticity fluctuations transport equation increases significantly and overcomes the production in a thin layer near the wall leading to a drastic diminution of the turbulent wall shear stress fluctuations.

1. Introduction

Development of efficient and feasible active drag control strategies is the new challenge of the wall turbulence research for the next decades. The socio-economic consequences of the drag reduction are becoming increasingly important for improving the performance of aircrafts and ships. New ideas are being developing in several teams around the world to achieve this delicate goal, by using either techniques based on the knowledge of near wall coherent structures and related physics [1,2] or by applying adaptive [3] or non-adaptive [4] non-linear control theory to the wall turbulence.

Optimal and suboptimal strategies are part of the second category. The goal of optimal control is to determine the actuation at the wall, which minimizes the total cost i.e., sum of the total shear stress and the cost for the intervention at the wall during a time interval T. This procedure is long and memory time consuming. Suboptimal control tries to pass beyond this shortcoming by proceeding at each time step. Suboptimal control techniques applied to the turbulent drag reduction problem have been introduced for a while [5,6], yet detailed analysis of the controlled flow field is curiously missing in the literature to our knowledge. Our objective is to revisit the suboptimal strategy through well-resolved direct numerical simulations at a comparatively higher Reynolds number in order to have a more profound physical insight into the turbulence response. A particular attention will be paid here to the structural alteration in the low buffer and viscous sublayers, especially through the vorticity transport mechanisms. It has to be emphasized that; as the title indicates only one-information strategies are concerned in this paper. The pre-determined cost function is based only on the streamwise component of the shear stress at the wall. There are substantial published works dealing with the performance development of suboptimal control techniques. For example Lee et al. [7] use two laws requiring, respectively, spatial information on the wall pressure over the entire wall and one component (spanwise) of the wall shear also over the entire wall. The aim here is not to develop more efficient suboptimal strategies nor low-cost control algorithms [8] the but to contribute to the understanding of the response of the wall turbulence to such an actuation, which has some striking facets, as it will be discussed. At last but not least, we have to indicate that, despite the considerable practical difficulties some researchers involve into well-designed experiments dealing with optimal or derivative strategies [9]. Thus, it is not impossible to have in the more or less next future real applications of active control that is worthwhile to be investigated in more details and from different points of view.

Suboptimal control has successively been applied to some flow control problems such as the separation control over a

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backward-facing step [10] or to vortex shedding [11] to give a few examples. Its application to drag reduction was however not convincing. The suboptimal control with cost function based directly on the wall shear stress results in less drag reduction than simpler ad-hoc strategies, despite the fact that it uses the information in the whole flow field. That is itself interesting and worth to be analyzed in detail, in particular regarding the wall structure modification under control that is missing in the literature. The wall shear stress is an accessible quantity from the measurements through micro wall gauges, while other strategies requiring spanwise variations of the fluctuating wall streamwise vorticity [3] is much harder to realize. Bewley et al. [4] indicated that "the minimization of a cost functional representing exactly the quantity of interest (drag) is *not necessarily* the *most* effective means of reducing the quantity of interest over the long term" without however establishing clearly the physical reasons. Thus, even in the optimal control, minimizing the cost function over global quantities such as the total turbulent kinetic energy or enstrophy reveals to be significantly more efficient than the cost function related directly to drag, even when the optimization horizon time is large. The one-information optimal control of drag gives only slightly better results than the best opposition control strategy ([4], Fig. 12). Thus the deficiency of the one-information suboptimal control based solely on the drag seems not to be entirely due to the optimization horizon. We will show that the main reason is presumably the lack of correlation of the local-instantaneous wall shear with the inner layer turbulence. To summarize, the main objectives of revisiting the one-information suboptimal control are:

(i) To report detailed data on the flow structure under the suboptimal control with the cost function solely based on the wall shear stress.
(ii) Analyze the control mechanism through the vorticity transport concept.
(iii) Compare and analyze the wall action under the suboptimal and ad-hoc strategies with the aim to depict why the former is less efficient than the latter.

2. Suboptimal control

Contrarily to the optimal control whose aim is to relaminarize the flow in a given time interval, the suboptimal strategy attempts to decrease at each time step the cost function. The latter is:

$$J(\phi) = \frac{k}{2\Gamma}\iint_w \phi^2\,\mathrm{d}S + \frac{1}{\Gamma}\iint_w \tau\,\mathrm{d}S \tag{1}$$

where τ is the shear at the wall whose area is denoted by Γ, ϕ is the action at the wall in the form of pinpoint blowing/suction distribution and k is a constant. The first integral above is clearly the energy expended to achieve the drag reduction. The control problem consists of determining the optimum ϕ at each time step. The state equation is the Navier–Stokes equation:

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \nu\frac{\partial^2 u_j}{\partial x_j^2} \tag{2}$$

where ν is the viscosity, u_i and x_i are, respectively, the instantaneous local velocity and coordinates and P denotes the pressure. Mixed notations will be used here for convenience, i.e., the streamwise (x_1), wall normal (x_2) and spanwise (x_3) directions will also be denoted, respectively, by x, y and z together with the corresponding velocity components $u(u_1)$, $v(u_2)$ and $w(u_3)$. The Eq. (1) is subject to the following boundary conditions at the wall, $x_2 = y = 0$:

$$\begin{aligned} u_1 &= 0 \\ u_2 &= \phi(x_1, x_3) \\ u_3 &= 0 \end{aligned} \tag{3}$$

The sensitivity of the cost function to the actuation modifications ϕ is measured through Fréchet derivatives as in classical non-linear control theory [12]. The variation of a functional $\xi(\phi)$, denoted by $\tilde{\xi}(\phi, \tilde{\phi})$ is given by:

$$\tilde{\xi}(\phi, \tilde{\phi}) = \lim_{\varepsilon\to 0}\frac{\xi(\phi + \varepsilon\tilde{\phi}) - \xi(\phi)}{\varepsilon} = \iint_w \frac{F\xi(\phi)}{F\phi}\tilde{\phi}\,\mathrm{d}S \tag{4}$$

where F stands for the Fréchet operator [13]. In practice, the Navier–Stokes equation is discretized in time and space, and the resulting operators are transformed through the Fréchet operator. Using a Crank–Nicholson scheme for the time discretization results for instance in the decomposition $Q^{n+1} + R^n = 0$ of (2) where Q^{n+1} and R^n regroup the terms at the times steps $n + 1$ and n. The resulting Fréchet transformation of (2) is consequently:

$$A\theta_i \equiv \theta_i + \beta_1\left(\frac{\partial\rho}{\partial x_i} + \lambda\delta_{i1} + \theta_j\frac{\partial u_i}{\partial x_j} + u_j\frac{\partial\theta_i}{\partial x_j}\right) - \beta_2\left(\frac{\partial}{\partial x_j}\frac{\partial}{\partial x_j}\theta_i\right) = 0 \tag{5}$$

where β_1 and β_2 are the coefficients resulting from the time discretization, and θ_i, ρ and λ are the Fréchet transforms of, respectively, u_i, the pressure fluctuations p' and the mean pressure gradient $\frac{\partial P}{\partial x_1}$, δ_{ij} standing for the Kronecker delta function. The last equation combined with the Fréchet transformation of the cost function:

$$\tilde{J}(\phi, \tilde{\phi}) = \iint_w \frac{\mathrm{D}J(\phi)}{\mathrm{D}\phi}\mathrm{d}S = \frac{k}{\Gamma}\iint_w \phi\tilde{\phi}\,\mathrm{d}S + \frac{1}{\Gamma}\iint_w \tilde{\tau}\,\mathrm{d}S \tag{6}$$

subject to the boundary conditions (3) allows the determination of the gradient $\frac{\mathrm{D}J}{\mathrm{D}\phi}$ from which the actuation at the next time step $n + 1$ is computed either by a conjugate gradient method $\phi^{n+1} = \phi^n - \alpha\left(\frac{\mathrm{D}J}{\mathrm{D}\phi}\right)^n$ or by a research of minima algorithm. By the introduction of an adjoint problem related to (5) and the convenient choice of its boundary conditions, $\frac{\mathrm{D}J}{\mathrm{D}\phi}$ can be related to the fluctuating adjoint pressure field at the wall. The adjoint operator A^* is defined by:

$$\langle A\theta_i, \varphi_i\rangle = \langle\theta_i, A^*\varphi_i\rangle + b \tag{7}$$

where φ_i is the adjoint of θ_i and b is a constant to be determined. The internal product $\langle f_1, f_2\rangle$ is the triple integral in the control volume V, $\langle f_1, f_2\rangle = \iiint_V f_1 f_2\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3$. Applying the operator (7) to the Eq. (5) results in:

$$A^*\varphi_i \equiv \varphi_i + \beta_1\left(\frac{\partial\pi}{\partial x_i} + \lambda\delta_{i1} + \varphi_j\frac{\partial u_j}{\partial x_i} - u_j\frac{\partial\varphi_i}{\partial x_j}\right) - \beta_2\left(\frac{\partial}{\partial x_j}\frac{\partial}{\partial x_j}\varphi_i\right) \tag{8}$$

Here λ and π stand, respectively, for the mean adjoint pressure gradient and fluctuating adjoint pressure field. The advantage of using an adjoint method is in the entire liberty of related boundary conditions choice. Taking conveniently the latter leads to:

$$\frac{\mathrm{D}J(\phi)}{\mathrm{D}\phi} = \frac{k}{\Gamma}\phi - \frac{\beta_1}{\beta_2\Gamma}\pi_w \tag{9}$$

relating the Fréchet variation of the cost function to the adjoint pressure field π_w at the wall. Thus both the Navier–Stokes equation (2) and its related adjoint Eq. (8) are resolved in time and space to determine π_w and the suboptimal distribution of blowing/suction actuation at the wall. The procedure is the same as used [5] with some subtle differences. We noticed for instance that the research of minima algorithm in the cost function at the time step n is particularly efficient when it is based on the gradient $\frac{\mathrm{D}J(\phi)}{\mathrm{D}\phi}$ computed at $n - 1$ and not n. Indeed, the wall shear stress, thereby the cost function cannot have proper information on the instantaneous change induced by a sudden variation in the boundary condition. A time lag of about $\Delta t^+ \approx \Delta y^{+2}$ in wall units (related to the shear