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Time-varying mesh stiffness calculation of spur gears with spalling defect

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ABSTRACT

Considering the effects of extended tooth contact (ETC), revised fillet-foundation stiffness under double-tooth engagement region, nonlinear contact stiffness and tooth spalling defect, an analytical model for time-varying mesh stiffness (TVMS) calculation of spur gears is established. In addition, the analytical model is also verified by comparing the TVMS under different spalling widths, lengths and locations with that obtained from finite element method. The results show that gear mesh stiffness decreases sharply with the increase of spalling width, especially during the single-tooth engagement; the spalling length only has an effect on the beginning and ending of gear mesh stiffness reduction; the spalling location can affect the range of gear mesh stiffness reduction, and the range will reduce when the spalling location is close to the addendum. This study can provide a theoretical basis for spalling defect diagnosis.

1. Introduction

Time-varying mesh stiffness (TVMS) of gear pairs has a great influence on the vibration of the gear systems, especially under tooth damage conditions such as spalling and crack. For the TVMS calculation, analytical methods based on the potential energy principle or deformation method are attracting increasing attention [1–13]. Considering Hertzian, bending and axial compressive energy, Yang and Lin [2] developed an analytical model to calculate the TVMS of a gear pair, and Yang's model [2] was further improved by Tian [3] and Wu [4] by introducing the shear energy. Wu's model [4] assumed the gear tooth to be a variable section cantilever beam clamped on the base circle, however, the actual tooth clamped on the dedendum circle. Aiming at this deficiency, Chaari et al. [5], Wan et al. [6], Liang et al. [7] and Ma et al. [8,9] developed different improved analytical models for calculating TVMS. Considering the effects of the gear tooth errors, Chen and Shao [10] developed a general analytical model to analyze the influences of tooth profile modification on TVMS. Taking the extended tooth contact into account, analytical methods for calculating TVMS are proposed by Ma et al. [11,12] and Yu and Mechefske [13]. Besides the analytical methods, many researchers also adopted finite element methods to calculate TVMS [8,11,12,14–18]. By applying the contact loads, Mohammed et al. [14] and Ma et al. [8] used 2D finite element (FE) models to calculate TVMS, and Zouari et al. [15] adopted 3D FE models to determine TVMS. Adopting contact elements to describe the mesh process of gears, Wang [16], Howard et al. [17], Jia and Howard [18] and Ma et al. [11,12] calculate the TVMS of gear pairs.

Based on the TVMS calculation methods, the TVMS of gear pairs with spalling has been investigated by many researchers. Chaari et al. [19] developed an analytical method to quantify the reduction of gear mesh stiffness due to spalling, and analyzed the effects of spalling widths and lengths on the TVMS. Based on Chaari's method, the effects of the spalling on the TVMS are evaluated by Ma et al. [20,21]. Shao et al. [22] developed a new TVMS calculation model considering the spalling and edge contact.

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Jiang et al. [23] proposed a new method for TVMS calculation of helical gears considering the effect of time-varying sliding friction and spalling defect. Jia and Howard [18] adopted a 3D FE model to analyze the effects of round spalling on the TVMS.

From the above literature analysis, it is clear that the effects of ETC are generally ignored during TVMS calculation of gear pairs with spalling based on the analytical method, and the analytical model needs to be verified. For these deficiencies, this study developed an improved TVMS calculation model for gear pairs considering the effects of the spalling, revised fillet-foundation stiffness and nonlinear Hertz contact stiffness and ETC, and verified the proposed model by comparing with FE method.

The frame of this paper is as follows: after this introduction, in Section 2, TVMS calculation method of spalled spur gears considering the effects of revised fillet-foundation stiffness, nonlinear contact stiffness and ETC are developed. Section 3 verifies the model by comparing the TVMS obtained from the proposed method with that obtained from FE method. Conclusions are presented in Section 4.

2. TVMS calculation method of spalled gears

2.1. TVMS calculation of spalled gears

In this paper, the shape of spalling is simulated by a rectangle groove [19] (see Fig. 1), and the groove is symmetrical about the mid-face of tooth. The spalling has the dimensions: w_s (spalling width) × l_s (spalling length a) × h_s (spalling depth). When the tooth contact occurs in the scope of the spalling, the cross-section S_i will change from a rectangle to a gib.

The gear tooth is modeled as a nonuniform cantilever beam, as shown in Fig. 2. In the figure, r_b denotes the radius of base circle, α_s denotes the pressure angle of the spalling starting position and β is the operating pressure angle. The mesh stiffness of single-tooth pair with spalling at meshing position j is written as follow [9]:

$$(k)_j = \frac{1}{\frac{1}{k_h}+\frac{1}{k_{t1}}+\frac{1}{k_{f1}}+\frac{1}{k_{t2}}+\frac{1}{k_{f2}}}, \tag{1}$$

where k_{ti} and k_{fi} (i = 1,2) are the stiffness of tooth and fillet-foundation, respectively, here, subscripts 1 and 2 denote the driving gear and driven gear; k_h denotes the local contact stiffness.

The stiffness of tooth k_{ti} can be expressed as:

$$k_{ti} = \begin{cases} \dfrac{1}{\frac{1}{k_{bi}}+\frac{1}{k_{si}}+\frac{1}{k_{ai}}} & (j \notin GD, i = 1,2) \\ \dfrac{1}{\frac{1}{k_{bsi}}+\frac{1}{k_{ssi}}+\frac{1}{k_{asi}}} & (j \in GD, i = 1,2) \end{cases}, \tag{2}$$

where GD is the scope of the spalling on tooth surface (see Fig. 2), j denotes meshing position, i (i = 1, 2) represents the driving gear and driven gear. The healthy tooth stiffness consists of the bending stiffness k_{bi}, shear stiffness k_{si} and axial compressive stiffness k_{ai}. For the spalled tooth, k_{bsi}, k_{ssi} and k_{asi} denote the bending stiffness, shear stiffness and axial compressive stiffness, respectively.

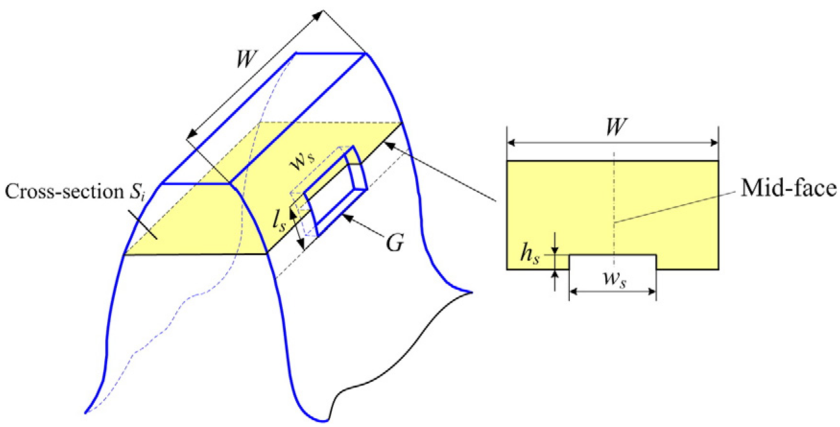

Fig. 1. Shape change of the cross-sections S_i due to spalling [19].