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Analytical model for mesh stiffness calculation of spur gear pair with non-uniformly distributed tooth root crack

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ABSTRACT

Gear tooth crack is likely to happen when a gear transmission train is working under excessive and/or long-term dynamic loads. Its appearance will reduce the effective tooth thickness for load carrying, and thus cause a reduction in mesh stiffness and influence the dynamic responses of the gear transmission system, which enables the possibility for gear fault detection from variations of the dynamic features. Accurate mesh stiffness calculation is required for improving the prediction accuracy of the dynamic features with respect to the tooth crack fault. In this paper, an analytical mesh stiffness calculation model for non-uniformly distributed tooth root crack along tooth width is proposed based on previous studies. It enables a good prediction on the mesh stiffness for a spur gear pair with both incipient and larger tooth cracks. This method is verified by comparisons with other analytical models and finite element model (FEM) in previous papers. Finally, a dynamic model of a gear transmission train is developed to simulate the dynamic responses when cracks with different dimensions are seeded in a gear tooth, which could reveal the effect of the tooth root crack on the dynamic responses of the gear transmission system. The results indicate that both the mesh stiffness and the dynamic response results show that the proposed analytical model is an alternative method for mesh stiffness calculation of cracked spur gear pairs with a good accuracy for both small and large cracks.

1. Introduction

Gear transmission train is widely employed in different areas, such as industrial machinery, automotive applications, locomotives, ships and airplanes, to transmit power and motions [1–3]. Its dynamic performance has attracted an intensive attention from the researchers and manufacturers due to its vital role in the mechanical transmission system. Excitations of a gear transmission train are always categorized into two types: external and internal excitations. The external excitations are usually from the fluctuations of the applied load and input operating speed [4–7], while the internal cyclic excitations, namely the time-varying mesh stiffness and transmission errors, are the inherited characteristics of a gear transmission system [2–9]. As one of the internal excitations, mesh stiffness appears to be time-varying due to variations of the number of teeth in mesh and the change of the contact position. How to capture it as accurately as possible is the goal of the related research works, which actuates the development of the numerical and analytical models for the mesh stiffness calculation.

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Finite element (FE) technique is frequently used in the gear mesh stiffness calculation [10–12]. However, the FE models of gear pairs need mesh refinement so as to capture the contact patterns and the detailed geometrical features, thus leading to an expensive computational cost. Fortunately, analytical models, as the alternative methods, could show good agreement with the results obtained by FE models with less computational time [13,14].

As early as in 1940s, Weber [15] developed an analytical model to calculate gear tooth deformations under load. In his model, the tooth compliance consists of three parts: the basic tooth deflection as a beam, the tooth deflection resulted from the fillet-foundation flexibility, and contact deflection of the teeth in mesh. Later, Cornell [13] extended Weber's model to include an improved fillet-foundation compliance analysis. Based on Weber's model, Chaari et al. [14] developed an analytical model to calculate the gear mesh stiffness. Recently, a so-called potential energy principle has been widely adopted ever since it was employed by Yang and Lin [16] to calculate the total mesh stiffness of spur gear pairs. By considering the effect of tooth shear deflection, this model was then refined by Tian [17] and Wu et al. [2], and further extended by Chen and Shao [3,19,20] with considering the fillet-foundation deflection [18]. Additionally, many research works attempted to extend these analytical models to the application in the calculation of gear mesh stiffness with tooth crack fault [2,3,14,15,19–28].

Presence of gear tooth crack will reduce the effective thickness of a gear tooth carrying the load. Consequently, it will cause a reduction in the mesh stiffness which is likely to generate some undesirable dynamic responses of the gear transmission system. The truth that tooth crack fault will change the dynamic responses enables the possibility of a vibration-based fault diagnosis for a gear transmission. Therefore, more precise methods of mesh stiffness calculation for cracked gear pairs are essential for better understanding the dynamic features with respect to the gear tooth crack failures. Li et al. [21,22] proposed an embedded-dynamic-fracture model to identify the gear meshing stiffness based on the measured gear angular displacement or transmission error and then predicted the gear fatigue crack propagation. Chaari et al. [14], based on Weber's model [15], developed a mesh stiffness calculation model for cracked spur gear tooth which is validated by FE analysis, where only the reduction of tooth thickness in the cracked area was considered.

Wu et al. [2] and Tian [17] developed an analytical model for mesh stiffness calculation of a cracked spur gear pair based on the potential energy principle. Then, Chen and Shao [3,19,20] developed an analytical calculation model to calculate the mesh stiffness of spur gears with non-uniformly distributed tooth root crack along tooth width. Up to now, the analytical mesh stiffness calculation models have been developed from 2-D to 3-D. Later, they [23] extended this model to be applied in the mesh stiffness calculation for cracked internal gear tooth. Pandya and Parey [24] used a slight curved crack propagation path to refine the total mesh stiffness calculation model based on the potential energy principle. Mohammed et al. [25] applied Chen's 3-D analytical model [3] to study the effect of the tooth crack propagating along both tooth width and crack depth simultaneously on the mesh stiffness. And further, the whole tooth plastic inclinations due to tooth bending damage are taken into consideration by Shao and Chen [26], and by Mark et al. [27,28] in their research work. Ma et al. [29] developed an improved method to calculate the mesh stiffness for cracked gear pairs considering the accurate transition curve at the tooth fillet. Liang et al. [30,31] presented the detailed formulas for spur gear mesh stiffness calculation when considering the deviation of the root circle from the basic circle. And recently, Ma et al. [32] made a detailed review on the dynamics of the cracked gear system where the mesh stiffness calculation models or methods with or without tooth crack were also involved, and they also pointed out the open problems about cracked gear dynamics for future research work.

Most of the previous papers considered the effect of tooth root crack by removing the materials within the area surrounded by the tooth profile, propagation path of the tooth root crack, and a line through the crack tip and parallel to the center line of the tooth. The remaining tooth segment is defined as the effective zone for load carrying. However, Mohammed et al. [33] found in their paper that the traditional way for removing the cracked tooth materials aforementioned could be only applied when the crack is small, saying less than 30% of the tooth thickness in their case studies, while it will cause a big deviation in the mesh stiffness when the crack is bigger. To solve this problem, they proposed a method, namely by replacing the line through the crack tip and parallel to the tooth center line with a parabolic curve through both crack tip and crack sided tooth tip based on the stress distribution obtained by FE analysis. This improved model enables more accurate calculation of gear mesh stiffness for both small and big cracks. However, the crack model they used was also 2-D by assuming the crack depth is constant along the whole tooth width.

In this paper, an improved analytical 3-D mesh stiffness calculation model for gear tooth root crack with its depth distributed non-uniformly along tooth width is developed. In this model, the area for removed materials with respect to the tooth root crack is defined as the “dead” zone surrounded by the tooth profile, propagation path of the tooth root crack, and the line connecting the crack tip and the crack sided tooth profile tip. This analytical model for both small and bigger crack depths is validated by comparison with the results obtained by FE model in Ref. [33]. Simultaneously, a parabolic curve through two known positions, namely the tooth tip and the crack tip, with its vertex at the tooth tip and its line of symmetry parallel to the center line of tooth has been proved to have similar accuracy with the proposed line-type method, indicating the usage of the line-type method to replace the parabolic curve is feasible. The proposed line-type method and the parabolic method are also applied to clarify the discrepancies between them and the previous model in Ref. [3] when they are employed to calculate the mesh stiffness of a spur gear pair with non-uniformly distributed crack depth along tooth width. The whole tooth plastically deformation is ignored in this paper.

This paper is organized as follows: reviews on the previous published literatures about the research work on mesh stiffness calculation and dynamic investigation of geared system with or without tooth root crack are carried out in introduction, which promotes the formation of the improved mesh stiffness calculation model of cracked spur gear pairs. Then, the improved analytical model is thoroughly introduced in Section 2 which is followed by the mesh stiffness calculation results in Section 3 to