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# A theoretical framework for Ångström equation. Its virtues and liabilities in solar energy estimation

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ABSTRACT

The relation between solar irradiation and sunshine duration was investigated from the very beginning of solar radiation measurements. Many studies were devoted to this topic aiming to include the complex influence of clouds on solar irradiation into equations. This study is focused on the linear relationship between the clear sky index and the relative sunshine proposed by the pioneering work of Ångström. A full semi-empirical derivation of the equation, highlighting its virtues and liabilities, is presented. Specific Ångström – type equations for beam and diffuse solar irradiation were derived separately. The sum of the two components recovers the traditional form of the Ångström equation. The physical meaning of the Ångström parameter, as the average of the clouds transmittance, emerges naturally. The theoretical results on the Ångström equation performance are well supported by the tests against measured data. Using long-term records of global solar irradiation and sunshine duration from thirteen European radiometric stations, the influence of the Ångström constraint (slope equals one minus intercept) on the accuracy of the estimates is analyzed. Another focus is on the assessment of the degradation of the equation calibration. The temporal variability in cloud transmittance (both long-term trend and fluctuations) is a major source of uncertainty for Ångström equation estimates.



## 1. Introduction

Two simple quantities related to the state of the sky are commonly used in the practice of solar energy estimation: (1) total cloud cover amount $C$, which represents the fraction of the celestial vault covered by clouds and (2) relative sunshine $\sigma$, which represents the time of sunshine expressed as the fraction of the maximum possible in a given time interval. While the total cloud cover amount is a direct measure for the state of the sky, the relative sunshine is an indirect indicator. During time, the statistical properties of these two quantities and their interconnection were extensively analyzed (e.g. the long-term trend in cloud amount and sunshine duration was studied in Ref. [1] based on measurements from 618 meteorological stations in China, and the fluctuation of the radiative regime was quantified by using the sunshine number in Ref. [2]).

Both parameters, total cloud cover amount and relative sunshine, are typically comprised in equations relating the solar irradiation at ground level to sunshine duration. The origin of such relations goes back to 1919 when Kimball [3] showed that the average daily global solar irradiance on a horizontal surface is nearly linearly related to the sunshine duration. In 1924, Ångström proposed a simple, empirically derived, linear equation between the monthly mean of daily global solar irradiation at ground level $\overline{H}$ and the monthly mean of daily sunshine duration $\overline{S}$ [4]:

$$\frac{\overline{H}}{\overline{H}_0} = 0.25 + 0.75\frac{\overline{S}}{\overline{S}_0} \tag{1}$$

where $\overline{H}_0$ represents the monthly mean of daily global solar irradiation in the hypothesis of clear sky and $\overline{S}_0$ represents the monthly mean of the day length. In Eq. (1) the ratio $\bar{\sigma} = \overline{S}/\overline{S}_0$ defines the monthly relative sunshine. The ratio $\bar{k}_{cs} = \overline{H}/\overline{H}_0$ is sometimes called the monthly *clear sky index*. $\bar{k}_{cs}$ quantifies the stochastic influence of clouds on the level of solar irradiation at ground level. Eq. (1) recovers the evidence that for a clear sky day $(\bar{\sigma} = 1)\,\overline{H} = \overline{H}_0$. For an overcast day $(\bar{\sigma} = 0)$, Eq. (1) suggest that $\overline{H}$ has a value of 25% of the solar irradiation in a perfectly clear sky day. A more general form of Eq. (1) was published later [5]:

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**Nomenclature**

| | | | |
|---|---|---|---|
| $C$ | total cloud cover amount | $H_{d0}$ | clear sky diffuse solar irradiation |
| $h$ | sun elevation angle | $J$ | julian day |
| $G$ | global solar irradiance | $k_{cs}$ | clear sky index |
| $G_b$ | beam solar irradiance | $k_t$ | clearness index |
| $G_d$ | diffuse solar irradiance | $S$ | sunshine duration |
| $G_{SC}$ | solar constant (1366.1 W m$^{-2}$) | $S_0$ | daylength |
| $H$ | global solar irradiation | $\chi$ | shadow number |
| $H_0$ | clear sky global solar irradiation | $\sigma$ | relative sunshine |
| $H_b$ | beam solar irradiation | $\tau_c$ | cloud transmittance |
| $H_{b0}$ | clear sky beam solar irradiation | $\xi$ | sunshine number |
| $H_d$ | diffuse solar irradiation | | |

$$\bar{k}_{cs} = \alpha + (1-\alpha)\bar{\sigma} \quad (2)$$

The empirical coefficient $\alpha$ incorporates all geographical and atmospheric dependencies. $\alpha$ ranges between 0.22 and 0.68 [6].

In order to avoid the difficulties arising in the endeavor of accurately estimating the clear sky solar irradiation, in 1940 Prescott replaced the reference $H_0$ by a deterministic quantity: the solar irradiation at extraterrestrial level, $\overline{H}_{ext}$ [7]. The ratio $k_t = \overline{H}/\overline{H}_{ext}$ defines the monthly *clearness index*. In general, an equation that relates the clearness index to relative sunshine (and/or other parameters) is referred to as an Ångström–Prescott equation, in recognition of the Ångström and Prescott pioneering work. The traditional form of the Ångström–Prescott equation is:

$$k_t = a + b \cdot \sigma \quad (3)$$

where $a$ and $b$ are empirical coefficients. Extensive work has been devoted during the last decades to the interpretation and to the improvement of the Ångström–Prescott equation. For instance, a literature survey and a case study are undertaken in [8] and a spatial modeling of the Ångström–Prescott parameters using data from 80 sites in China, in order to extend the applicability of the model is done in [9].

There are far fewer studies on the Ångström equation than on the Ångström–Prescott equation. However, in time, different developments were proposed for Eq. (2). Yang et al. [10] developed a linear correlation in which global solar irradiation is expressed as the sum of the diffuse and beam components, each of them being expressed independently by an equation similar to Eq. (2). A relevant physical interpretation of the Ångström equation and a nonlinear correlation between the solar irradiation and relative sunshine are reported in [11]. An updated version of the model was proposed thirteen years later [12]. Several other nonlinear equations similar to Eq. (2), but as power functions of the relative sunshine, were proposed (e.g. [13] for Romania, [14] for Turkey).

Although the Ångström–Prescott equation, in its various forms, is currently used worldwide in estimating the solar irradiation (for example in 2013 in China [15] or in 2015 in Tunisia [16]), the subject of this paper comes back to the original equation of Ångström. A motivation is presented next.

Firstly, Eq. (2) has long been only of academic importance, its practical application being limited by the difficulties in estimating the clear sky solar irradiation. The advances made in modeling the clear sky solar irradiance in the last decades made rising above this limitation possible (see e.g. the comprehensive review of 54 clear sky models reported in [17]). Available codes, as Simple Model of the Atmospheric Radiative Transfer of Sunshine (SMARTS) [18] developed by Gueymard [19], allows an accurate estimation of solar irradiance anywhere in the world. Furthermore, the online services, such as NASA Surface Meteorology and Solar Energy (SMSE) [20], provide estimates of the clear sky solar irradiation based on satellite observations. This idea of fulfilling the conditions for turning back to the Ångström equation is also considered by Polo et al. [21].

Secondly, it is generally agreed that the local climates are changing rapidly under the influence of pollution and anthropogenic aerosols. There are some aerosol–cloud interactions known to have indirect effect on climate: (i) the Twomey effect, which describes the increase of cloud albedo due to the changing concentrations of cloud condensation and ice nuclei, possibly from anthropogenic generated aerosols. This leads to a decrease of solar radiation on ground, for thin and moderately thick clouds [22]; (ii) the Albrecht effect which describes how cloud condensation nuclei, also possibly generated by anthropogenic pollution, may increase cloud lifetime, delay precipitation formation, and increase cloud lifetime and subsequently total cloud cover [23]. These changes cause some uncertainty on whether the calibration of the empirical equations for solar irradiation, including the Ångström equation, degrades over time. Since the extinction of solar radiation due to clouds is more significant than that due to any other atmospheric constituent, this paper focuses on the relation between the changes in the properties of the clouds and the changes in the amount of collectable solar energy. In particular, the change in the clouds transmittance rooted in the Ångström equation is being analyzed. As further demonstrated in Section 2, the empirical coefficient $\alpha$ in Eq. (2) *naturally* isolates the full information on the average of the clouds transmittance. Therefore, the Ångström equation can be a potential tool for cloud characterization.

This paper reports a self-consistent derivation of the Ångström equation, highlighting the assumption required to obtain it and its inherent limitations. As a novelty, separate equations are derived for the beam and the diffuse components of the global solar irradiation. Section 2 is devoted to this topic. The clearly stated hypotheses for the Ångström equation derivation, the statistical analysis on the accuracy of the equation for beam solar irradiation, the concept of shadow number are elements that ascribe originality to this theoretical framework for the Ångström equation. Section 3 presents original results obtained by numerical investigation of the Ångström equation performance: (1) Aiming to substantiate the theoretical result, a quantitative assessment on the accuracy of the beam solar irradiation estimates in different aggregated time intervals is presented; (2) The influence of the Ångström constraint in fitting Eq. (2) to data is analyzed in comparison to standard linear regression; (3) An investigation of the variation in time and space of the cloud transparency retrieved from Ångström equation is performed. From this, useful information on the calibration of the Ångström equation is extracted. All the numerical investigations were carried out using the mathematical software MathCAD [24].