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# Mathematical modeling and control of plate fin and tube heat exchangers

Dawid Taler *

*Cracow University of Technology, Faculty of Environmental Engineering, ul. Warszawska 24, 31-155 Cracow, Poland*

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ABSTRACT

The aim of the study is to develop a new method for numerical modeling of tubular cross-flow heat exchangers. Using the method proposed in the paper, a numerical model of a car radiator was developed and implemented in a digital control system of the radiator. To evaluate the accuracy of the numerical method proposed in the paper, the numerical model of the car radiator was compared with an analytic model. The proposed method based on a finite volume method and integral averaging of gas temperature across a tube row is appropriate for modeling of plate fin and tube heat exchangers, especially for exchangers in which substantial gas temperature differences in one tube row occur. The target of control is to regulate the number of fan revolutions per minute so that the water temperature at the heat exchanger outlet is equal to a set value. Two control techniques were developed. The first is based on the numerical model of the heat exchanger developed in the paper while the second is a digital proportional–integral–derivative control. The first control method is very stable. The digital proportional–integral–derivative controller becomes unstable when the water volume flow rate varies considerably. The developed techniques were implemented in digital control system of the water exit temperature in a plate fin and tube heat exchanger. The measured exit temperature of the water was very close to the set value of the temperature if the first method was used. The experiments show that the proportional–integral–derivative controller works also well but becomes frequently unstable if the water flow rate varies.



## 1. Introduction

Plate fin and tube heat exchangers are widely used as economizers in steam power boilers, air-conditioning and heat pump coils, convectors for home heating, waste-heat recovery systems for gas turbines, and water–air heat exchangers in power plant cooling towers [1]. They are also commonly used as the air finned coolers, automobile radiators and heater cores in automobiles, which are used to heat the air within the passenger compartment [2]. Automotive air-conditioning and climate control systems are discussed in detail by Daly [3]. Zhang et al. [4] evaluated theoretically the performance of a finned tube evaporator designed to recover the exhaust waste heat from an internal combustion engine. Finned or plate fin and tube heat exchangers can be manufactured by electrical or laser welding of the fins to the tube or by expanding the holes in continuous plate fins. The fins can also be extruded directly from the tube wall metal, especially in tubes made of aluminum [1]. Recently, a variety of techniques of enhancing air and water side heat transfer in cross-flow tube heat exchangers has been developed [5]. Bayat et al. [6] found that the cam-shaped tubes in the staggered arrangement perform better compared with the circular tubes. The effect of fin pitch and number of tube rows on the air side performance of herringbone wavy fin and tube heat exchangers was studied experimentally by Wongwises and Chokeman [7]. Wu et al. [8] experimentally analyzed two novel fin-tube surfaces with punched longitudinal vortex generators to demonstrate significant heat transfer enhancement.

The most commonly methods used in the design and performance calculations of the cross-flow tube heat exchangers, is the LMTD method (logarithmic mean temperature difference) or the $\varepsilon$-NTU method (exchanger effectiveness $\varepsilon$ – number of transfer units NTU) [9]. The both methods are simple and extensively used in the engineering practice. On the basis of these methods, Wakui and Yokoyama [10] developed a steady-state model of a shell and tube heat exchanger for the online model-based performance monitoring. However, the key assumptions used in the two methods such as constant heat capacity of liquids or constant heat

* Tel.: +48 12 6283026; fax: +48 12 6282866.
*E-mail address:* dtaler@pk.edu.pl

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**Nomenclature**

| | |
|---|---|
| $A$ | area ($m^2$) |
| $A_{cin}, A_{co}$ | inner and outer cross section area of the oval tube ($m^2$) |
| $A_f$ | heat transfer area of the fin ($m^2$) |
| $A_{mf}$ | area of the bare tube between two adjacent fins ($m^2$) |
| $A_m$ | mean surface of the tube, $A_m = (A_o + A_{in})/2$ |
| $A_{min}$ | minimum free flow frontal area on the air side ($m^2$) |
| $c$ | specific heat (J/(kg K)) |
| $c_p\|_0^T$ | mean specific heat at constant pressure at the temperature interval with the limits 0 and $T$ |
| $A_{in}, A_o$ | inner and outer surface of the bare tube ($m^2$) |
| $d_h$ | hydraulic diameter of air flow passages (m) |
| $d_{min}$, $d_{max}$ | minimum and maximum outer diameter of the oval tube, respectively (m) |
| $d_r$ | hydraulic diameter on the liquid side, $4A_{in}/P_{in}$ (m) |
| $e$ | control error (K) |
| $h$ | convective heat transfer coefficient (W/($m^2$ K)) |
| $h_a, h_w$ | the air- and water-side heat transfer coefficient (W/($m^2$ K)) |
| $h_o$ | effective heat transfer coefficient considering fin efficiency based on the outer surface area of the bare tube (W/($m^2$ K)) |
| $k$ | thermal conductivity (W/(m K)) |
| $k_t$ | thermal conductivity of the tube material (W/(m K)) |
| $(k)$ | iteration number |
| $K_d$ | derivative gain (s) |
| $K_i$ | integral or reset gain (1/s) |
| $K_p$ | controller gain |
| $L_{ch}$ | length of the heat exchanger (m) |
| $\dot{m}$ | mass flow rate (kg/s) |
| $\dot{m}_a$ | air mass rate in the automobile radiator (kg/s) |
| $\dot{m}_g$ | gas mass flow rate per tube (kg/s) |
| $\dot{m}_l$ | liquid mass flow rate per tube (kg/s) |
| $\dot{m}_w$ | water mass rate in the automobile radiator (kg/s) |
| $N_g, N_l$ | number of transfer units for the air and water side, respectively |
| $Nu_w$ | water-side Nusselt number, $h_w d_r/k_w$ |
| $Nu_a$ | air-side Nusselt number, $h_a d_h/k_a$ |
| $p_1$ | pitch of tubes in plane perpendicular to flow (height of the fin) (m) |
| $p_2$ | pitch of tubes in the direction of flow (width of the fin) (m) |
| $P_{in}, P_o$ | inner and outer perimeter of the oval tube, respectively (m) |
| $Pr$ | Prandtl number, $\mu c_p/k$ |
| $\dot{Q}$ | heat transfer rate (W) |
| $Re_a$ | air-side Reynolds number, $w_{max} d_h/\nu_a$ |
| $Re_w$ | liquid-side Reynolds number, $w_w d_r/\nu_w$ |
| $s$ | fin pitch (m) |
| $t$ | time (s) |
| $T$ | temperature (°C or K) |
| $T'_{am} T''_{am}$ | mean inlet and outlet temperature of the air (°C) |
| $T'_g, T''_g$ | gas temperature before and after the tube (°C) |
| $T'_w, T''_w$ | water inlet and outlet temperature, respectively (°C) |
| $T''_{w,1}, T''_{w,2}$ | water outlet temperature from the first and the second tube row in the upper pass, respectively (°C) |
| $T_{wm}$ | outlet temperature of the water after the first pass (°C) |
| $T''_{w,set}$ | target (preset) value for the water temperature at the outlet of the heat exchanger (°C) |
| $T''_{w,meas}$ | measured water temperature at the outlet of the heat exchanger (°C) |
| $u$ | controller output |
| $\bar{u}$ | steady-state controller output |
| $U_o$ | overall heat transfer coefficient that is referred to the outer surface area of the bare tube (W/($m^2$ K)) |
| $\dot{V}_w$ | water volume flow rate at the inlet of the heat exchanger (L/h or $m^3$/s) |
| $w_0$ | average frontal flow velocity (air velocity before the heat exchanger) (m/s) |
| $w_{max}$ | mean axial velocity in the minimum free flow area (m/s) |
| $x, y, z$ | Cartesian coordinates |
| $x^+$ | dimensionless coordinate, $x^+ = x/L_{ch}$ |
| $y^+$ | dimensionless coordinate, $y^+ = y/p_2$ |

*Greek symbols*

| | |
|---|---|
| $\delta_f$ | fin thickness (m) |
| $\delta_t$ | tube wall thickness (m) |
| $\Delta t$ | time step in digital PID control |
| $\Delta t_c$ | time step in model-based control |
| $\mu$ | dynamic viscosity (Pa s) |
| $\eta_f$ | fin efficiency |
| $\nu$ | kinematic viscosity ($m^2$/s) |
| $\rho$ | fluid density (kg/$m^3$) |
| $\tau_d$ | derivative time (s) |
| $\tau_i$ | integral or reset time (s) |

*Subscripts*

| | |
|---|---|
| $a$ | air |
| $g$ | gas |
| in | inner |
| $l$ | liquid |
| $p$ | at constant pressure |
| $w$ | water |

*Superscripts*

| | |
|---|---|
| + | dimensionless |
| — | mean |
| ′ | inlet |
| ″ | outlet |

transfer coefficient on the length of the heat exchanger are often not met in real heat exchangers.

Recently, CFD modeling has been used for thermal and hydraulic calculations of plate fin and tube heat exchangers. Extensive CFD simulations of the performance of fin-and-tube heat exchangers were carried out numerically. Most of these studies deals with the influence of geometrical parameters such as tube diameter, tube and fin thickness, tube and fin pitch on the flow and thermal characteristics. Tao et al. [11] simulated three-dimensional laminar flow and heat transfer in three stage heat exchangers with plane and slit fins using the finite volume method. A plane fin and tube heat exchanger was modeled by Zhou and Catton [12] based on volume averaging theory. A commercial finite volume method-based code, CFX, was used to model the convective heat transfer in a representative elementary volume for fin and tube heat exchanger. Applying a finite element simulation in COMSOL, a fast running numerical model for prediction of pressure drop in a fin tube heat exchanger was developed by Rezk and Forsberg [13]. Taler and Ocłoń [14] correlated air-side heat transfer results based on the CFD modeling for plate fin and oval tube heat exchangers. The influence of fin-to-tube thermal contact resistance on the reduction of the transfer rate was studied in the work [15]. Čarija