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Application of decomposition method and inverse prediction of parameters in a moving fin

Rohit K. Singla, Ranjan Das *

School of Mechanical, Materials and Energy Engineering, Indian Institute of Technology, Ropar, Punjab 140001, India

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ABSTRACT

The application of the Adomian decomposition method (ADM) is extended to study a conductive–convective and radiating moving fin having variable thermal conductivity. Next, through an inverse approach, ADM in conjunction with a binary-coded genetic algorithm (GA) is also applied for estimation of unknown properties in order to satisfy a given temperature distribution. ADM being one of the widely-used numerical methods for solving non-linear equations, the required temperature field has been obtained using a forward method involving ADM. In the forward problem, the temperature field and efficiency are investigated for various parameters such as convection–conduction parameter, radiation–conduction parameter, Peclet number, convection sink temperature, radiation sink temperature, and dimensionless thermal conductivity. Additionally, in the inverse problem, the effect of random measurement errors, iterative variation of parameters, sensitivity coefficients of unknown parameters are investigated. The performance of GA is compared with few other optimization methods as well as with different temperature measurement points. It is found from the present study that the results obtained from ADM are in good agreement with the results of the differential transformation method available in the literature. It is also observed that for satisfactory reconstruction of the temperature field, the measurement error should be within 8% and the temperature field is strongly dependent on the speed than thermal parameters of the moving fin.

1. Introduction

Extended surfaces increase the heat transfer rate because of the surface area increment [1]. The governing equation for heat transfer phenomena can be represented by differential equations and the desired effect in the form of temperature field, heat transfer rate, efficiency, etc., can be calculated by solving the differential equation alongwith relevant boundary conditions. The assumptions of constant thermo-physical properties reduce the mathematical complexity of the differential equation, but, in reality the thermo-physical properties vary with temperature and material [2,3]. Many studies involving different numerical methods for solving heat transfer problems in fins are available in literature [4–8]. The thermal performance of a circular convective–radiating porous fin has been analyzed using the least squares method (LSM) and fourth order Runge–Kutta method [9]. Such type of methods have been also implemented for longitudinal convective–radiating, ceramic-based materials (SiC and Si_3N_4), porous fin with various fin profiles [10]. Moreover, effects of various parameters, such as porosity, Darcy number, Rayleigh number, and Lewis number on the fin efficiency have been also investigated for fully wet circular porous fins [11].

Many industrial processes such as extrusion, hot rolling, glass fiber drawing and casting are modeled as moving fin, where the heat transfer from the extruded products, and rolled sheet to the surroundings occurs in continuous motion [12–14]. In addition to this, the fins are also set in motion when an automobile is in moving condition. From the literature it is observed that some good studies involving various numerical techniques dealing with the moving fin have been reported earlier. Using Runge–Kutta–Fehlberg method based on maple 13 package, Aziz and Lopez [15] have analyzed processing time of a moving sheet with temperature-dependent thermal conductivity. Aziz and Khani [16] have also studied a moving fin with variable thermal conductivity using homotopy analysis method. For a conductive–convective and radiating moving fin with variable thermal conductivity, Torabi et al. [17] demonstrated the application of the differential transformation method (DTM). The wavelet collocation method and Haar wavelet method were used to investigate the effects of various

* Corresponding author. Address: School of Mechanical, Materials and Energy Engineering Indian Institute of Technology, Ropar Nangal Road, Rupnagar, Punjab 140001, India.

E-mail address: ranjandas81@gmail.com (R. Das).

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Nomenclature

A_n, B_n, C_n, D_n, E_n	Adomian polynomials
A	area of cross section, m^2
a	dimensionless thermal conductivity
c	temperature at insulated end of fin (constant)
c_p	specific heat of the fluid, J kg^{-1} K^{-1}
h	heat transfer coefficient, W/m^2 K
k	thermal conductivity, W/m K
L	instantaneous length of fin, m
ξ_{x^*}	second order differential operator
$\xi_{x^*}^{-1}$	two-fold integral operator
N_c	convection–conduction parameter
N_r	radiation–conduction parameter
P	perimeter of fin, m
Q	heat transfer rate, W
Pe	Peclet number
T	temperature, K
U	speed of moving fin, m s^{-1}
X	axial distance measured from the insulated end
X^*	non-dimensional axial distance

Greek symbols

α	thermal diffusivity of the fluid, m^2 s^{-1}
β	thermal conductivity variation parameter, K^{-1}
ε	emissivity of fin material
θ	dimensionless temperature
ρ	density of the fluid, kg m^{-3}
σ	Stefan–Boltzmann constant
η	fin efficiency

Subscripts

a	ambient temperature, K
b	base of fin
c	convection
r	radiation

parameters for a continuously moving convective and radiative fin by Singh et al. [18] and Ravi Kanth and Uday Kumar [19], respectively.

Now a days, analytical approaches such as differential transformation method (DTM), homotopy analysis method (HAM), variational iteration method (VIM), Adomian decomposition method (ADM) are gaining interest for nonlinear problems in heat and mass transfer. Kundu and Lee [20] have applied DTM for wet fin with temperature-dependent thermal conductivity and heat transfer coefficient. A convective porous fin with temperature-dependent internal heat generation has been studied using LSM, DTM and CM (collocation method) by Hatami et al. [21]. Das and Ooi [22] have implemented decomposition method for a rectangular fin with temperature-dependent heat transfer coefficient. A fin-shaped micro channel heat sink with Cu–water nanofluid has been investigated by Hatami and Ganji [23]. Panda et al. [24] have been applied HAM over a rectangular wet fin by considering nonlinear effects both in the governing equation and the boundary conditions.

For highly nonlinear problems, the Adomian decomposition method (ADM) is one of the efficient and widely-used numerical techniques which was introduced and developed by Adomian [25]. The main advantage of ADM is that it generally provides rapid convergence and can be directly applied to equations containing linear or nonlinear, homogeneous or inhomogeneous, terms. To maintain higher accuracy level of the solution and to reduce computational work, ADM is very much effective [26]. The Adomian polynomials are little bit difficult to evaluate and this is the only demerit of ADM [27]. However, DTM, which is a semi-analytical technique [28] does not require evaluation of any polynomial and its solution is found to possess either equal or less accuracy than the corresponding ADM solution [29,30]. ADM is also extensively applied for solving various heat transfer problems in fins [31–36]. However, most of the studies concentrate to predict the thermal behavior either in the form of temperature field or efficiency from *a priori* knowledge of thermo-physical properties of the fin. These types of analyses are known as forward (direct) problems [37]. The situation becomes different and interesting when the problem is to identify some suitable thermo-physical parameters when some desired temperature distribution is required to be achieved. Problems of such kind are known as inverse problems, which are mathematically ill-posed [38] and multiple solution may also exist which can result in same output [39]. Similar to forward problems, many inverse problems dealing with heat transfer problems are available in literature [40–44].

The available literature indicates that ADM is yet not applied to study moving fins. In addition to this, for a moving fin, the inverse problem is also not available. Therefore, the present work satisfies following two objectives,

(a) Applying ADM to study the heat transfer problem in a moving fin and investigate the influence of various parameters on the temperature and efficiency distributions.
(b) To solve an inverse problem for simultaneously estimating heat transfer coefficient, thermal conductivity and the speed of moving fin for satisfying a given temperature field.

In addition to the above-mentioned objectives, finally a sensitivity analysis is also carried out for finding out the most influencing parameter contributing to the temperature field. Below we discuss the formulation and solution methodology for the present problem.

2. Mathematical formulation

Let us consider the geometry of a moving fin with temperature-dependent thermal conductivity properties with details shown in Fig. 1. The heat transfer coefficient, h and the surface emissivity, ε are assumed to be constant throughout the surface of the fin. The fin is assumed to be conductive–convective and radiating. Various parameters of the fin are cross-sectional area, A_c, perimeter, P, and the speed of the fin, U. One of the ends of the fin is subjected to

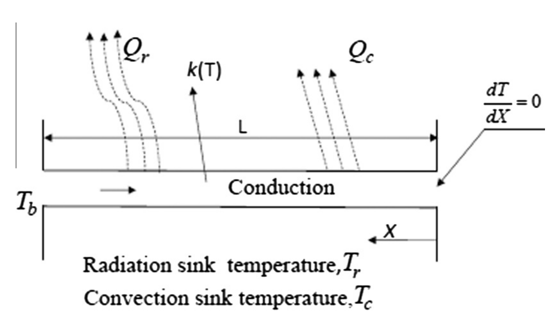

Fig. 1. Geometry of moving fin [14].