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# Temperature-dependent conductivity in forced convection of heat exchangers filled with porous media: A perturbation solution

Maziar Dehghan, Mohammad Sadegh Valipour *, Seyfolah Saedodin

*Faculty of Mechanical Engineering, Semnan University, Semnan, Iran*

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ABSTRACT

Effects of variation of the thermal conductivity on forced convection in a parallel-plates channel heat exchanger occupied by a fluid saturated porous medium are investigated analytically based on the perturbation methods. Walls of the channel are kept at a constant heat flux. Thermal conductivity of the medium is assumed to be a linear function of temperature (due to moderate radiation heat transfer in cellular foams or temperature dependent conductivity of the material). The Brinkman–Forchheimer-extended Darcy model for the flow field is used. Relations representing the temperature profile and Nusselt number as functions of porous medium shape parameter and thermal conductivity variation parameter are derived. Obtained Nusselt number and temperature profile are studied parametrically. No analytical investigation based on a variable conductivity approach for Brinkman–Forchheimer-extended Darcy model has been previously performed. Results show that a linear increase in the thermal conductivity of the medium results in a semi-linear increase in the Nusselt number.

## 1. Introduction

The porous media can be naturally formed (rocks, sand beds, sponges, woods) or fabricated (catalytic pellets, insulations). Heat and fluid flow through porous materials occur in a large number of industrial applications such as oil and gas flow in reservoirs, geothermal energy harvesting, drying, storage of absorbed solar energy, water and mineral migration, catalytic converter for air pollution reduction, and filtering [1,2]. Recent applications of porous media include the flow of liquids in biological and physiological processes, micro-fluidics, solar energy, geothermal energy, and cooling of turbine blades in the hot portion of a turbo-expander [3–7]. Porous medium insertion in a heat-transferring domain is a way to increase the heat transfer rate. Today, other ways of heat transfer rate enhancement together with the implementation of porous media could be pioneering in the thermal performance optimizations [8–12].

Perturbation is a powerful technique in solving equations, especially non-linear equations governing the heat and fluid flows in porous media. Vafai [13] analyzed the effects of variable porosity and inertial forces on convective flow and heat transfer in the boundary layer of an impermeable wall filled with porous media. He used both numerical simulation and the perturbation (matched asymptotic expansion) method and found that the variable porosity is important near the wall in packed beds. Kaviany [14] solved the Brinkman–Darcy momentum and energy equations for forced convection inside the iso-flux channel analytically. Later, Vafai and Kim [15] proposed closed form solutions to the Brinkman–Forchheimer–extended Darcy momentum and energy equations for forced convection inside an iso-flux channel filled with porous media. Herwig and Koch [16] used the regular perturbation technique to solve the momentum and energy equations of a highly porous matrix placed in the boundary layer of an impermeable wall. Hooman and Ranjbar-Kani [17] used perturbation methods (straight expansion and WKB) to solve the Darcy–Brinkman momentum and energy equations for a pipe fully filled with porous media. He proposed dimensionless velocity and temperature distributions at iso-flux boundary conditions. Hooman [18] applied both straight expansion and matched asymptotic expansion methods to solve the Darcy–Brinkman–extended Forchheimer momentum and energy equations for a channel fully filled with porous media for the case of iso-flux boundary condition. He showed that the Forchheimer number has the least effect on the dimensionless velocity and temperature distributions among other parameters. Recently, Dehghan et al. [19,20] investigated the local thermal non-equilibrium (LTNE) condition for porous media bounded by parallel-plates or tubes using the two-equation energy model analytically. They proposed a new dimensionless number

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* Corresponding author at: PO 35131-19111, Semnan, Iran. Tel.: +98 2333383352; fax: +98 2333654122.

*E-mail addresses:* m-dehghan@semnan.ac.ir, dehghan.maziar@gmail.com (M. Dehghan), msvalipour@semnan.ac.ir (M.S. Valipour), s_sadodin@semnan.ac.ir (S. Saedodin).

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## Nomenclature

| | |
|---|---|
| $c_p$ | specific heat at constant pressure (J kg$^{-1}$ K$^{-1}$) |
| $C_F$ | inertial constant |
| $Da$ | Darcy number, $K/H^2$ |
| $F$ | Forchheimer number |
| $G$ | negative of the applied pressure gradient in flow direction (Pa m$^{-1}$) |
| $H$ | half of the channel gap (m) |
| $K$ | permeability of the medium (m$^2$) |
| $k$ | effective thermal conductivity of the medium (W m$^{-1}$ - K$^{-1}$) |
| $k_f$ | thermal conductivity of fluid phase (W m$^{-1}$ K$^{-1}$) |
| $k_m$ | effective conductivity of medium at bulk mean temperature (W m$^{-1}$ K$^{-1}$) |
| $k_s$ | thermal conductivity of solid phase (W m$^{-1}$ K$^{-1}$) |
| $k_w$ | effective conductivity of the medium at the wall temperature (W m$^{-1}$ K$^{-1}$) |
| $M$ | viscosity ratio |
| $n$ | mathematical power |
| $Nu_m$ | Nusselt number based on $k_m$ |
| $Nu_w$ | Nusselt number based on $k_w$ |
| $O$ | order of magnitude |
| $q''_w$ | heat flux at the wall (W m$^{-2}$) |
| $s$ | porous media shape parameter |
| $T$ | temperature (K) |
| $T_m$ | bulk mean temperature (K) |
| $T_r$ | temperature variation parameter |
| $T_w$ | wall temperature (K) |
| $u$ | dimensionless velocity |
| $u^*$ | velocity (m s$^{-1}$) |
| $\widehat{u}$ | normalized velocity |
| $u^*_m$ | mean velocity (m s$^{-1}$) |
| $x^*$, $y^*$ | dimensional coordinates (m) |
| $y$ | dimensionless coordinate |

*Greek letters*

| | |
|---|---|
| $\beta_R$ | Rosseland mean extinction coefficient (m$^{-1}$) |
| $\varepsilon$ | linear proportionality multiplier of the variable thermal conductivity model |
| $\theta$ | dimensionless temperature |
| $\mu$ | fluid viscosity (kg m$^{-1}$ s$^{-1}$) |
| $\mu_{eff}$ | effective viscosity in the Brinkman term (kg m$^{-1}$ s$^{-1}$) |
| $\sigma$ | Stefan–Boltzmann coefficient (W m$^{-2}$ K$^{-4}$) |
| $\rho$ | fluid density (kg m$^{-3}$) |
| $\phi$ | porosity of the medium |

*Subscripts*

| | |
|---|---|
| 0,1 | coordinate identifier and constant value in conductivity profile |
| *eff* | effective value |
| *f* | fluid phase |
| *K* | permeability |
| *m* | mean |
| *s* | solid phase |
| *w* | wall |

representing the intensity of the LTNE condition based on a perturbation analysis. Also, they obtained the normalized velocity and dimensionless temperature distributions inside the channel. They showed that the normalized velocity is independent of the Forchheimer effect by the order of $s^{-3}$ ($\widehat{u} = \widehat{u}(y,s) + O(s^{-3})$), where $s$ denotes the porous medium shape parameter given by Eq. (9). The porous medium shape parameter has a high value in practical porous media [2,19].

Alazmi and Vafai [21] investigated on variants within the porous media transport models numerically. Four major categories in modeling the transport processes through porous media, namely constant porosity, variable porosity, thermal dispersion, and local thermal nonequilibrium, were analyzed in their study. They concluded that in general the variances have more influence on the velocity field than the temperature field and Nusselt. Nield and Kuznetsov [22] studied the effects of variation of permeability and thermal conductivity on fully developed forced convection in fluid saturated porous media analytically. They used a linear model for the change of permeability and thermal conductivity with transverse direction. Their results demonstrated that the effect of permeability variation is that an above average permeability near the walls leads to an increase in Nusselt number. The Nusselt number was not always a monotonic function of the conductivity variation. They completed their study by a two-equation model for the energy equation at the non-equilibrium condition. Nield and Kuznetsov [23] found that the effect of local thermal non-equilibrium is significant when the solid conductivity is greater than the fluid conductivity. Sundaravadivelu and Tso [24] added effects of viscosity variation to the work of Nield and Kuznetsov [22]. They approximated the viscosity as a linear function in the transverse direction. Their results revealed that the Nusselt number decreases with the increase of permeability ratio of a layered medium for strong viscosity variations, which cannot be captured if a constant viscosity model is assumed. The above mentioned studies were at the fully developed condition. However, effects of the viscosity variation in a homogenous medium are confined to a small region near the walls. Nield and Kuznetsov [25] extend their model (Nield and Kuznetsov [22]) for a thermally developing flow between parallel-plate channels. They used a modified Graetz methodology to define the developing temperature profile for the Darcy's law of motion.

All studies mentioned in the previous paragraph concerned the case of heterogeneous porous media in which the permeability and conductivity vary because of the structure of media. On the other hand, effects of temperature dependent properties could be important. Hooman and Gurgenci [26] investigated the effect of temperature-dependent viscosity on the developed forced convection in a porous duct of rectangular cross-section analytically. They assumed Darcy flow model, uniform heat flux at the walls, and an inverse-linear viscosity–temperature relation. They showed that the Nusselt number increases with reduction of the temperature-dependent viscosity. Hooman and Gurgenci [27] investigated effects of temperature-dependent viscosity on Bénard convection in an enclosure filled with a porous medium numerically. They discussed on the effects of property variation on the natural convection. Nield and Kuznetsov [28] investigated a combined conductive–convective–radiative process in a channel occupied by a saturated cellular porous medium. They assumed Darcy–Brinkman model, a constant molecular thermal conductivity, and a temperature-dependent radiative conductivity. They obtained an analytical solution to the case of variable conductivity for Darcean flow. Nield and Kuznetsov [28] showed that the Nusselt number increases at the case of variable conductivity. Based on an extension to work of Dehghan et al. [29], they [30] numerically investigated a combined convection–radiation heat transfer inside a micro-heat exchanger filled with a porous medium in the slip-flow regime using a temperature dependent thermal conductivity. They showed that the temperature jump phenomenon