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Modeling and forecasting monthly movement of annual average solar insolation based on the least-squares Fourier-model

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ABSTRACT

Solar insolation is one of the most important measurement parameters in many fields. Modeling and forecasting monthly movement of annual average solar insolation is of increasingly importance in areas of engineering, science and economics. In this study, Fourier-analysis employing finite Fourier-series is proposed for evaluating monthly movement of annual average solar insolation and extended in the least-squares for forecasting. The conventional Fourier analysis, which is the most common analysis method in the frequency domain, cannot be directly applied for prediction. Incorporated with the least-square method, the introduced Fourier-series model is extended to predict its movement. The extended Fourier-series forecasting model obtains its optimums Fourier coefficients in the least-square sense based on its previous monthly movements. The proposed method is applied to experiments and yields satisfying results in the different cities (states). It is indicated that monthly movement of annual average solar insolation is well described by a low numbers of harmonics with approximately 6-term Fourier series. The extended Fourier forecasting model predicts the monthly movement of annual average solar insolation most fitting with less than 6-term Fourier series.

1. Introduction

As energy consumption has been exponentially globally increased, energy is vital for social and economic sustainable development and environmental security. Energy modeling & forecasting [1–6] and management is increasingly important for its effect on economic prosperity, social sustainable development and environmental security in the future, which are linked to a vide variety of industries, such as industrial production and agriculture, population and education, health and life, emission and environmental protection [7]. Solar energy is a clean, renewable and sustainable source of energy. It has the potential to meet global energy demands while reducing greenhouse gas emissions to mitigate global climate change and minimize harm to the environment. Solar insolation, the total amount of solar energy received at one particular location of the earth during a specified time period, has been one important measurement parameter in many fields. For calculating yearly energy harvesting of grid-connected PV (photovoltaic) systems, the authors of [8] presented a new classification approach, which are classified in both direct and indirect methods. For the minimum heat loss from the non-evacuated solar receiver of a solar collector, the authors of [9] presented a numerical study based on the CFD (computational fluid dynamics) model. For optimizing the essential components' geometry in a SCPP (solar chimney power plant), the authors of [10] presented a computational study by employing a CFD software called ANSYS-CFX to evaluate and improve the flow qualities inside the SCPP. On evaluating the potential of applying DSWHs (domestic solar water heating systems), the authors of [11] indicated that its yearly savings in electrical energy was about 1316–1459 kW h/year and its annual greenhouse-gas emission per house was expected to be reduced by 27,800 tCO2. By employing the statistical non-linear regression technique, the authors of [12] tested six models for forecasting daily insolation in 4 Iranian cities. For calculation of monthly average daily total radiations, the authors of [13] presented one investigation based on measured radiation data on horizontal surface at Bahawalpur. For design of a solar energy system involves assessing local availability of solar energy and its relationship to local climate, the authors of [14] found that high correlation existed in summer sunny days, but very little correlation in winter. By using one photovoltaic silicon solar meter, the authors of [15] investigated the intensity of solar insolation at Enugu, Nigeria. For estimation of surface solar insolation, the authors of [16] employed an ANN-based (artificial neural network) model. The author of [17] presented one insolation-oriented

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model of photovoltaic module by employing Matlab/Simulink. On the problem of influence of extraterrestrial forcing on the Earth's climate, the authors of [18] presented one approach for unifying orbital, solar and lunar forcing based on their common control of the Earth's latitudinal insolation gradient (LIG). To assess the solar potential through windows and building openings, the authors of [19] developed a simple optical device. For installations of wind and solar power plants, based on wind speed and insolation period data, the authors of [20] presented one investigation on similarity, feasibility and numerical analysis of 75 cities in Turkey.

Monthly movement of annual average solar insolation, which may be influenced by various factors, is also a time-series. Since all the factors influencing its movement are implicitly enclosed in its time-sequence, there are two main goals included in its time-series analysis [21]: (a) try to discover the nature of the phenomenon depicted described by the observed sequence, and (b) try to build a model to predict its future movement based on the time-series variable. Techniques employed in time-series analysis mainly include the classical time-series analysis, methods in frequency domain and artificial intelligence. The classical time-series analysis is a standard technique in statistics, which includes 3 well-known classical models called the AR (autoregressive) model, the MA (moving average) model and a combination of the two called the ARMA model. This technique analyzes time-series under the important assumption that the underlying stochastic process is stationary, and then the process could adequately describe be the lower moments of its probability distribution. Artificial neural network (ANN) is one of the most popular and accepted models in artificial intelligence. It has been widely used in many fields and reported with good performance for its nonlinear learning ability. However, besides its time consuming and the rule-of-thumb choices in establishing its network structure, two major risks the called over-fitting and under-fitting (i.e., excessive or less training data approximation) still exist in employing ANN models, which will increase its errors in the out-of-sample forecasting.

Fourier analysis technology [22–24] is the most common analysis method in the frequency domain. It is a stable and powerful usability tool for time-series analysis. However, the conventional Fourier analysis can be not directly used for prediction. In this study, we try to employ the extended Fourier-series model in the least-squares [25] for predicting monthly movement of annual average solar insolation. Contributions in this study are: (1) Introduce a finite Fourier-series model for evaluating monthly movement of annual average solar insolation. (2) Present a forecast method based on the extended Fourier-series model in the least-squares for predicting monthly movement of annual average solar insolation. The forecast model is built by seeking its optimum Fourier coefficients in the least-square sense based on its previous monthly movements. The obtained Fourier coefficients associated with their harmonics in the built model is used for forecasting. (3) Show workability of the proposed method. Result analysis indicates that the proposed method yields satisfying results in the experiments of evaluating and forecasting for monthly movements of annual average solar insolation.

2. Fourier-series model for evaluating monthly movement of annual solar insolation

Fourier analysis grew from the study of Fourier-series, which deals with the basic concept that signals can be approximately described by a sum of sinusoids at different frequencies [22–24]. A Fourier-series is an expansion of a continue-time periodic signal in terms of an infinite sum of oscillating functions (namely sines and cosines, or complex exponentials). The Fourier-series in the trigonometric form for describing a continue-time signal $s(t)$ of periodicity T is by [22–24,25],

$$s(t) = a_0 + \sum_n (a_n \cos(n \cdot \cot \omega_1 \cdot t) + b_n \sin(n \cdot \omega_1 \cdot t)) \tag{1}$$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} s(t)\mathrm{d}t, \quad \omega_1 = \frac{2\pi}{T}$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} s(t)\cos(n\cdot\omega_1 t)\mathrm{d}t, \quad b_n = \frac{2}{T}\int_{t_0}^{t_0+T} s(t)\sin(n\cdot\omega_1 t)\mathrm{d}t$$

where a_0, a_n and b_n, are called the Fourier coefficients. n is number of harmonics and ω represents frequency and a_0 is the constant term (also called the DC component: a zero frequency component) ($n = 0$).

With sampling interval $\Delta T = \frac{T}{N}$ in the interval $[t_0, T + t_0]$ (usually set $t_0 = 0$), we have N samples for the continuous-time signal $s(t)$ and $s(k\ \Delta T)$ is the kth sample. Put $k = 1,\ldots,N$ (or $k = 0\ldots(N-1)$), the Fourier-series can be approximated based on the discrete-time samples $s(k\ \Delta T)$, which is also the nature of discrete Fourier-series [25]:

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} s(t)\mathrm{d}t \doteq \frac{1}{N\Delta T}\sum_{k=1}^{N} s(k\Delta T)\Delta T = \frac{1}{N}\sum_{k=1}^{N} s(k\Delta T)$$

$$\begin{aligned} a_n &= \frac{2}{T}\int_{t_0}^{t_0+T} s(t)\cos(n\cdot\omega_1 t)\mathrm{d}t \doteq \frac{2}{N\Delta T}\sum_{k=1}^{N} s(k\Delta T) \\ &\quad \times \cos\left(n\cdot\frac{2\pi}{N\Delta T}k\Delta T\right)\Delta T \\ &= \frac{2}{N}\sum_{k=1}^{N} s(k\Delta T)\cos\left(n\frac{2\pi}{N}k\right) \end{aligned} \tag{2}$$

$$\begin{aligned} b_n &= \frac{2}{T}\int_{t_0}^{t_0+T} s(t)\sin(n\cdot\omega_1 t)\mathrm{d}t \doteq \frac{2}{N\Delta T}\sum_{k=1}^{N} s(k\Delta T) \\ &\quad \times \sin\left(n\cdot\frac{2\pi}{N\Delta T}k\Delta T\right)\Delta T \\ &= \frac{2}{N}\sum_{k=1}^{N} s(k\Delta T)\sin\left(n\frac{2\pi}{N}k\right) \end{aligned}$$

Set $\Delta T = 1$, we have,

$$\begin{aligned} & s(k\Delta T) \to s(k) = a_0 + \sum_n (a_n \cos(n\tfrac{2\pi}{N}k) + b_n \sin(n\tfrac{2\pi}{N}k)) \\ & a_0 \doteq \tfrac{1}{N}\sum_{k=1}^{N} s(k) \\ & a_n \doteq \tfrac{2}{N}\sum_{k=1}^{N} s(k)\cos(n\tfrac{2\pi}{N}k), \ b_n \doteq \tfrac{2}{N}\sum_{k=1}^{N} s(k)\sin(n\tfrac{2\pi}{N}k) \end{aligned} \tag{3}$$

where $s(k)$ is the discrete time function, and N is called the discrete-time period.

To evaluate monthly movement of annual average solar insolation $S(t)$, we extend its movement to be annual periodic or quasi-periodic that put $N = 12$ for each year has 12 months. From Eqs. (1) and (3), the Fourier-series model for evaluating monthly movement of annual average solar insolation $S(t)$ is presented as follows:

$$S(t) \doteq a_0 + \sum_{n=1}^{M}\left(a_n \cos\left(n\frac{2\pi}{N}t\right) + b_n \sin\left(n\frac{2\pi}{N}t\right)\right)(N = 12) \tag{4}$$

where $a_0 = \frac{1}{N}\sum_{k=1}^{N} S(k), a_n = \frac{2}{N}\sum_{k=1}^{N} S(k)\cos(n\frac{2\pi}{N}k), b_n = \frac{2}{N}\sum_{k=1}^{N} S(k) \sin(n\frac{2\pi}{N}k)$, n is the nth term of the Fourier series, k the sequential number of monthly solar insolation from 1 to 12 (months), $S(k)$ the kth actual average solar insolation in the kth month, t the local time, and it may be continuous or discrete between 1 and 12