Energy Conversion and Management 78 (2014) 14–23

Contents lists available at ScienceDirect

Energy Conversion and Management

journal homepage: www.elsevier.com/locate/enconman

Effects of streamwise conduction on thermal performance of nanofluid flow in microchannel heat sinks

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ARTICLE INFO

Article history:
Received 12 September 2013
Accepted 23 October 2013
Available online 16 November 2013

Keywords:
Exponential wall heat flux
Microchannel heat sink
Nanofluid
Streamwise conduction

ABSTRACT

Effects of streamwise conduction on the thermal performance of nanofluid flow in microchannel heat sinks under exponentially decaying wall heat flux are investigated. By employing the first-law principles, models with and without streamwise conduction term in the energy equation are developed for hydrodynamically fully-developed flow. Closed-form solutions are obtained and the analysis emphasizes on the details of discrepancy induced by streamwise conduction between the two models on the heat transport characteristics in nanofluids. The effects of the variations of Peclet number and nanoparticle volume fraction on the thermal characteristics of nanofluid flow in microchannel heat sinks are analyzed and discussed. Due to the tremendous increase in the effective thermal conductivity, the streamwise conduction effect is justified to be more significant in the nanofluid compared to its base fluid. The significance of the streamwise conduction which is prevalent in low-Peclet-number flow is greatly amplified when the volume fraction of nanoparticle is increased. At low Peclet number, the contribution of the streamwise conduction in nanofluid is found to be more than twofold of that in its base fluid. The effect of streamwise conduction on the nanofluid flow in microchannel heat sink is significant albeit not dominant particularly for low Peclet number and high nanoparticle volume fraction of the nanofluid.

1. Introduction

The unprecedented growth in electronics technologies, especially the miniaturization of the communication and computing devices, poses a challenging problem in the thermal management of such devices. Micro-scale heat transfer becomes a topical subject and the microchannel heat sink has manifested itself as one of the promising solutions to the thermal problem in light of its high thermal performance. To further enhance the thermal performance of the microchannels, the choice of working fluid has been a key issue. The addition of suspended nano-scale solid particles in the base fluid has been reported to be able to enhance the heat transfer characteristic of the conventional fluids. Choi [1] first coined the term "nanofluid" for this type of fluid which exhibits anomalous increase in the effective thermal conductivity even with a small volume fraction of nanoparticle suspension. Reviews of studies on the heat transfer characteristics of nanofluids have been well documented [2–8]. Numerous theoretical studies have been performed to investigate various effects related to the hydrodynamic and thermal characteristics of nanofluids [9–18]. Unlike the micro-sized particles, the use of the ultrafine nanoparticles is free from clogging problem in microchannel flows. Therefore, nanofluid is a promising candidate of innovative working fluid in microchannel flows for the sake of heat transfer enhancement.

Most of the existing analytical studies on microchannel heat sink neglected the effect of streamwise conduction despite the fact that this effect is justified to significantly affect the heat transport rate in proximity to the entrance region of the fluid flow for conventional size channels [19–21]. The streamwise conduction effect appears to be pronounced at the entrance region but diminishes further away from the entrance [22]. For small Peclet number, the characteristic time of convection and conduction is comparable and hence the streamwise conduction becomes indispensable. It was reported that significant error can be generated by neglecting the streamwise conduction for a Peclet number value of 10 [23]. The Nusselt number correlations have been recommended for low-Peclet-number flow by taking into account the effect of the streamwise conduction [24]. Due to the fact that the internal flows in micro-scale devices are typically characterized by finite Peclet numbers, the incorporation of the effect of streamwise conduction is a necessity in the thermal analysis of microchannel flow. The inclusion of streamwise conduction generally involves the Graetz-type problems which are inherently complicated due to the presence of the non-self-adjoint eigenvalues and the non-orthogonal eigenfuctions [25]. To evade the mathematical complexity, the streamwise conduction effect in microchannel flow can be characterized analytically by averaging the fluid temperature over the

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http://dx.doi.org/10.1016/j.enconman.2013.10.061

Nomenclature

Br′	modified Brinkman number
C_k	thermal conductivity ratio
C_μ	viscosity ratio
c_p	specific heat (J kg^{-1} K^{-1})
D	inner diameter of microchannel (m)
d	diameter of nanoparticle (m)
k	thermal conductivity (W m^{-1} K^{-1})
Kn	Knudsen number
L	length of the microchannel (m)
p	pressure (Pa)
Pr	Prandtl number
q	heat rate (W)
q''	heat flux (W m^{-2})
Q	total heat transfer rate for the entire length of microchannel (W)
r	radial coordinate (m)
r_0	inner radius of microchannel (m)
Re	Reynolds number
T	temperature (K)
$\overline{T}$	mean temperature (K)
u	fluid velocity (m s^{-1})
$\bar{u}$	mean fluid velocity (m s^{-1})
x	longitudinal coordinate (m)
X	Dimensionless longitudinal coordinate

Greek symbols

β	exponent of wall heat flux variation (m^{-1})
$\widehat{\beta}$	dimensionless exponent of wall heat flux variation
ϕ	nanoparticle volume fraction
μ	dynamic viscosity (N s m^{-2})
ρ	fluid density (kg m^{-3})
λ	molecular mean free path (m)
γ	heat capacity ratio as defined in Eq. (23)
ξ	aspect ratio as defined in Eq. (18)
$\overline{\theta}$	dimensionless mean temperature

Subscripts

cond	of streamwise conduction
conv	of convection
eff	effective
f	of base fluid
in	of total heat input
L	value at position L
nf	of nanofluid
p	of nanoparticle
vd	of viscous dissipation
w	value at wall
x^*	value at position x^*
0	value at the entrance
1	of Model 1
2	of Model 2

microchannel cross section, leading to a one-dimensional energy equation for the incompressible fluid flow with constant physical properties [26]. The presence of streamwise conduction increases the difference between the wall-fluid temperature and reduces the Nusselt number value within the entrance section. Similar results were obtained by employing the fin approach with a one-dimensional analysis on the streamwise conduction where the fluid temperature distributions exhibit significant discrepancy with those of neglecting the streamwise conduction effect when the Peclet number is sufficiently small for different channel aspect ratio and fluid-to-wall thermal conductivity ratio [27]. The incorporation of streamwise conduction in microchannel flow can also be modeled by employing a porous-medium model which was solved numerically using finite difference method [28]. Comparison was done with the analytical solutions by neglecting the streamwise conduction. Numerical surrogate optimization analysis with an evolutionary algorithm using commercial CFD software has also been performed to study the effect of the streamwise conduction in microchannel heat sinks [29].

Judging from the tremendous increase in the effective thermal conductivity of nanofluids, it is instructive to take the effect of streamwise conduction into account in the evaluation of the heat transfer rate of nanofluid flow in microchannels. In the existing literature, none of the studies deals explicitly with the effect of streamwise conduction on nanofluid flow in microchannels, in spite of the anomalous increase in the effective thermal conductivity of nanofluids which in turn contributes significantly to the streamwise conduction. Such effect is essential in understanding the mechanism of heat transfer processes and predicting the heat transfer rate of nanofluid flow in micro-scale channels. The present study, a basic investigation in filling this gap, emphasizes details of the streamwise conduction heat transfer rates of nanofluid flow in a circular microchannel with exponential wall heat flux thermal boundary condition. Closed-form solutions for the temperature distributions are obtained by solving the governing equations analytically. With the variations of the Peclet number and the nanoparticle volume fraction, the role of the streamwise conduction heat transfer is investigated and its contribution is compared with that of the convection heat transfer. The effect of streamwise conduction on the total heat transfer of nanofluid is scrutinized and compared with that of its base fluid. The underlying physical significance of the streamwise conduction in nanofluid flow in microchannel heat sink is discussed.

2. Mathematical formulation

Nanofluid poses distinct thermal behavior from the conventional fluid associated with its three distinguished main transport properties: thermal conductivity, heat capacity, and viscosity. The addition of the ultra-fine nanoparticles into the base fluid changes these transport properties prominently, enhancing the thermal performance of the nanofluid. The effective thermal conductivity of nanofluid is higher than that of its base fluid with suspension of a small volume fraction of nanoparticles. In the present study, the nanofluid of water-alumina is chosen and its effective thermal conductivity k_{eff} can be predicted using the correlation taking into account the Brownian motion-induced convection from multiple nanoparticles as [30].

$$k_{\text{eff}} = C_k k_f, \tag{1}$$

where C_k is a constant coefficient defined as

$$C_k = \left(1 + A\text{Re}_b^m \text{Pr}^{1/3}\phi\right) \frac{\kappa(1+2\alpha) + 2 + 2\phi[\kappa(1-\alpha) - 1]}{\kappa(1+2\alpha) + 2 - \phi[\kappa(1-\alpha) - 1]}. \tag{2}$$

In Eq. (2), $\kappa = k_p/k_f$ is the thermal conductivity ratio of the thermal conductivity of the particle k_p to the thermal conductivity of the base fluid k_f, $\text{Pr} = c_{p,f}\mu_f/k_f$ is the Prandtl number, ϕ is the nanoparticle volume fraction, $\alpha = 2R_b k_f/d$ is defined as the Biot number of the particle, with d is the diameter of nanoparticle and R_b is the