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A numerical study of viscous dissipation effect on non-Newtonian fluid flow inside elliptical duct

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ABSTRACT

Laminar heat transfer inside duct with elliptical cross section, subjected to uniform wall temperature is studied by taking into account the viscous dissipation. The temperatures distributions are evaluated numerically by using a dynamic Alternating Direction Implicit method (*dADI*). Nusselt number (*Nu*) is presented graphically for various Brinkman number (*Br*) and aspect ratio for a non-Newtonian fluid described by the power law model. The results obtained showed a good agreement with those found in the literature for fluid flow in circular cross section ducts and in elliptical cross section without viscous dissipation effects. It is shown that in the fully developed region and for $Br \neq 0$, Nusselt number has a fixed asymptotic value independent of Brinkman number (*Br*). In the thermally developing region, it is observed a single fixed point independent of heating or cooling condition which the numerical value is equal to the asymptotic Nusselt number. Another relevant feature is that in the fully developed region, the Nusselt number increases with the aspect ratio.

1. Introduction

Knowledge of the steady state heat transfer forced convection problem inside ducts is important in many engineering applications, such as for the heat exchanger devices encountered in nuclear power systems and air conditioning equipment. The investigations on heat transfer to thermally developing, hydrodynamically developed forced convection inside ducts of simple geometries such as circular tube or parallel plates have been performed by numerous researchers. There exist other studies on the extensions to the Graetz problem. The important extensions are the consideration of viscous dissipation [1–3], the non-Newtonian fluid flow [4–6], both non-Newtonian fluid flow and viscous dissipation through parallel plates [7–9], circular ducts [10–14], and the fully developed incompressible fluid flowing inside tubes of arbitrary cross section shapes [15–17]. In the literature, only a limited amount of works is available on laminar forced convection inside a straight duct of elliptical cross section. This is the reason why in this work, the effects of viscous dissipation of non-Newtonian fluids in thermally developing flow inside elliptical duct is performed. Someswara et al. [18] examine the problem of laminar forced convection heat transfer in short elliptical duct. The Lévêque theory of linear velocity profile near the wall is used. Correlations for the local and average Nusselt number and wall temperature were obtained. Cain et al. [19] presented an experimental study of turbulent heat transfer fluid flowing inside elliptical duct section by considering different aspect ratios. A reduction in the overall heat transfer rate was found in the lower turbulent Reynolds number range ($Re < 25000$). Richardson [20] studied the laminar flow of a Newtonian fluid with fully developed velocity profile in an elliptical duct with constant wall temperature. An approximate Lévêque solution is used to obtain the developing temperature field in the duct for high Graetz number.

In Ref. [21], Bhatti analysed the laminar flow in the entrance region of an elliptical duct. A closed-form analytical solution for a Newtonian fluid is developed. The analysis is based on the Karman–Pohlhausen integral method. In another study [22], by using the complex variables method, Bhatti examined the heat transfer in the fully developed region of elliptical duct with uniform wall heat flux. Universal curves in fully developed temperature distribution and Nusselt number were presented. Abdel Wahed et al. [23], performed experiments on laminar flow and heat transfer in an elliptical duct with aspect ratio $\beta/\alpha = 0.5$. The results were obtained for two conditions at the wall, namely constant wall temperature and a linear distribution of temperature. Huang et al. [24] developed a mathematical model to determine the properties of air flow in the elliptical duct. The nonlinear governing equations are transformed into hyperbolic type and the characteristics method is applied. Velusamy and Grag [25] using a numerical method based on a control volume studied the fully developed mixed convection problem in vertical ducts of elliptical cross-section. The duct wall is subjected to a uniform axial heat flux. Nusselt and crit-

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Nomenclature

a, b	major and minor ellipsis semi-axis (m)
Br	Brinkman number
c_p	specific heat (J kg^{-1} K^{-1})
D_h	hydraulic diameter (m)
J	Jacobian determinant, Eq. (5.5)
k	fluid conductivity (W m^{-1} K^{-1})
m	consistency factor
n	flow behaviour index
Nu	local Nusselt number
P	perimeter of the elliptical contour (m)
T	temperature (K)
T_0	inlet temperature (K)
Re	Reynolds number
U	dimensionless velocity, Eq. (3.1)
u, v	dimensionless elliptical coordinates
V	velocity, Eq. (1.2) (m s^{-1})
x, y, z	coordinates axes (m)
X, Y, Z	dimensionless coordinates

Greek symbols

α, β	dimensionless ellipsis semi-axes
α^*	dimensionless focal distance
ρ	fluid density (kg m^{-3})
θ	dimensionless temperature, Eq. (5a)
μ	fluid apparent viscosity, Eq. (1.3)
Γ	cross section contour
Ω	cross section domain

Subscripts

av	average
w	wall

ical Rayleigh number are presented for a wide range of duct aspect ratios and Rayleigh number. They found that during mixed convection, fluid with higher axial velocity exists around the foci of the elliptical cross-section, leading to substantial heat transfer enhancement in this region of the duct. Sakalis et al. [26] analysed the thermally developing flow in elliptic ducts with axially variable wall temperature distribution. Numerical results obtained with ADI scheme indicate that the friction factor increases and the Nusselt number decreases when aspect ratio decreases. Maia et al. [27] used a generalised integral transform technique to solve analytically the thermally developing problem of power law non-Newtonian laminar fluid flow inside elliptical duct. The results are presented for different aspect ratios by neglecting the effect of viscous dissipation. In order to enhance the heat transfer, numerous designs for industrial heat exchangers involving a bank of tubes were applied. Among many types of heat exchangers, those constructed of elliptic tubes have been used in many industrial applications. Using elliptic tubes with eccentricity of 0.5, Rocha et al. [28] reported that the thermal efficiency has been enhanced about 18% and the pressure drop reduced of 25% comparatively with the circular tubes. This result is supported by Matos et al. [29] and Tao et al. [30].

In the open literature, the internal forced convection in noncircular geometries are not widely investigated comparatively with the classical geometries such as parallel plates or circular tube. The investigations of non-Newtonian laminar fluid flow in elliptical duct by taking account the viscous dissipation effect are not available. The main objective of the present research is to investigate numerically the effect of viscous dissipation on non-Newtonian fluids flowing inside elliptical duct. The numerical method used was validated with an analytical solution for the particular case neglecting the viscous dissipation. The effects of Brinckman number, power law index and aspect ratio on the temperature distribution and Nusselt number were analysed.

2. Problem statement and mathematical formulation

Let us consider a non-Newtonian fluid that steadily flows within infinite elliptical cross section duct ($-\infty \leqslant z \leqslant \infty$) subjected to a constant wall temperature T_w as the boundary condition. The fluid is subjected to uniform inlet temperature T_0 at $z \to -\infty$ as shown in Fig. 1a. The rheological behaviour of non-Newtonian fluids considered here can be described by the power law model also known as the Ostwald–de Waele model. Moreover, the flow is considered laminar, hydrodynamically fully developed and thermally developing. The thermophysical properties of the fluid are assumed to be thermo-independent. It is known that the thermal conductivity of non-Newtonian fluids is influenced by the shear rate when the rate values are important. This assumption can be supported by the studies of Picot et al. [31] who experimentally showed that the thermal conductivity of polyethylene liquid was enhanced of 10% for a shear rate 400 s^{-1}. Loulou et al. [32] performed measurements of thermophysical properties of non-Newtonian fluid (aqueous polymer solution of 0.2% Carbopol 940) and then predict a variation on thermal conductivity of 6.5% for a shear rate range [0–20 s^{-1}]. The amount of this variation decreases as the temperature increases. Also, as reported by Lamsaadi et al. [33], the viscous dissipation can be neglected only for polymer solutions weakly or moderately concentrated such as aqueous solutions of CMC (carboxymethylcellulose) but not for polymer solution highly concentrated or polymer melts, where the viscous frictions are significant. The Brinkman number, considered as a criterion showing the relative importance of viscous dissipation is taken here in a reasonable range ($-1 < Br < 1$) [2,3,7–12]. The energy equation according to cartesian coordinates is given by:

$$\rho c_p V(x,y)\frac{\partial T(x,y,z)}{\partial z} = k\left[\frac{\partial^2 T(x,y,z)}{\partial x^2} + \frac{\partial^2 T(x,y,z)}{\partial y^2}\right] + \mu\left[\left(\frac{\partial V(x,y)}{\partial x}\right)^2 + \left(\frac{\partial V(x,y)}{\partial y}\right)^2\right] \quad (1.1)$$

The velocity profile $V(x,y)$ for a fully developed flow of the power law fluid in a duct of elliptical cross section is given by [27] as:

$$V(x,y) = \frac{3n+1}{n+1}\left[1 - \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^{\frac{n+1}{2n}}\right]V_{av} \quad (1.2)$$

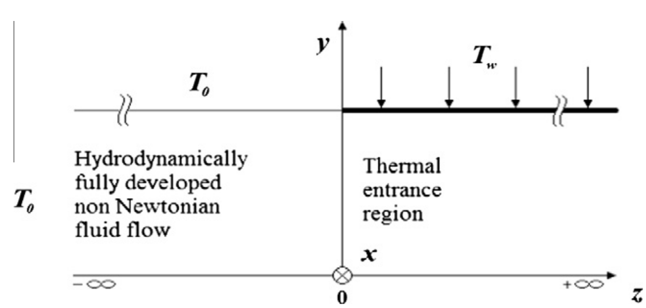

Fig. 1a. Geometrical configuration.