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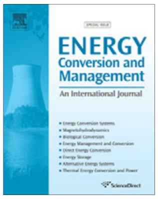

Optimal short-term operation schedule of a hydropower plant in a competitive electricity market

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ABSTRACT

This paper presents a dynamic programming model to solve the short-term scheduling problem of a hydropower plant that sells energy in a pool-based electricity market with the objective of maximizing the revenue. This is a nonlinear and non-concave problem subject to strong technical and strategic constraints, and in which discrete and continuous variables take part. The model described in this paper determines, in each hour of the planning horizon (typically from one day to one week), both the optimal number of units in operation (unit commitment) and the power to be generated by the committed units (generation dispatch). The power generated by each unit is considered as a nonlinear function of the actual water discharge and volume of the associated reservoir. The dependence of the units' efficiency and operating limits with the available gross head is also accounted for in this model. The application of this model to a real hydropower plant demonstrates its capabilities in providing the operation schedule that maximizes the revenue of the hydro plant while satisfying several constraints of different classes. In addition, the use of this model as a supporting tool to estimate the economic feasibility of a hydropower plant development project is also analyzed in the paper.

1. Introduction

The Spanish electricity market is organized around a short-term wholesale pool-based market where every day generating companies, consumers and energy services companies submit bids for selling or buying energy for the next 24 h. Rules established for this market are supposed to promote the economic efficiency in such a way that the search for the maximum benefit, carried out by these agents, lead to the maximum social benefit [1]. The generation capacity, among other factors, conditions the bidding strategy of a generating company. In this sense, generating companies can be divided into two categories: price-maker and price-taker. This paper considers a hydropower plant that sells energy in the day-ahead electricity market and whose influence on the market clearing process may be assumed negligible due to its small size in comparison with the entire power system. In this environment, the price clearing process may be assumed exogenous [2] to the hydro plant scheduling and the energy prices can be taken as fixed input parameters for the optimal operation schedule search process. Bidding strategies considering the effect of the plant selling bids on the market clearing procedure [3] are therefore outside the scope of this paper.

Several methods have been used to study the short-term scheduling of hydro and hydrothermal energy systems as it can be seen in [4]; from basic heuristic procedures, such as the old, but still in use, *peak shaving method* [5] to modern intelligent artificial techniques, such as neural networks [6] or evolutionary algorithms [7]. In this sense, it is important to point out that, although the approach of the short-term scheduling model may vary slightly depending on the electricity market environment, the plant/reservoir regulation capability and whether the energy system is hydro- or thermal-based, essentially, most scheduling models found in technical references are applicable to some extent in a broad range of cases.

Classical mathematical programming methods have been widely used for this purpose, namely: linear programming [8], nonlinear programming [9], mixed-integer linear programming [10] and mixed integer quadratic programming [11]. Since the beginning of the 1980s, decomposition techniques have also been applied, as it is the case of Benders decomposition technique [12] and, mainly during the last decade, Lagrangian relaxation technique [13]; this has been recognized in the technical literature as the most promising procedure to solve short-term hydrothermal coordination problems [14] (both Benders decomposition and Lagrangian relaxation techniques are explained in detail in [15]).

Optimal control based algorithms have also been used to solve hydroelectric resources scheduling problems [16]. Along with linear programming, dynamic programming has probably been the

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Nomenclature

The notation used throughout the paper is presented next:

f_c	conversion factor (0.0036 $hm^3/h/m^3/s$)
f^{t_w}	maximum revenue function for week t_w
$F(x_{ik})$	maximum power output from state x_{ik} (kW)
$I(i,j,t)$	partial revenue obtained in stage t according to trajectory i–j (€)
$\bar{I}(i,t)$	maximum revenue accumulated at the beginning of stage t from state i (€)
I^a	maximum expected annual revenue (€)
K	set of generating units of the hydro plant
p_{ijk}	power generated by unit k according to trajectory i–j (kW)
$p(i,j,t)$	plant power output during stage t in trajectory i–j (kW)
q	available flow to be optimally distributed among all available units (m^3/s)
$q(i,j,t)$	flow discharged during stage t according to trajectory i–j (m^3/s)
$q_k^{\max}$	maximum water discharge through unit k
$q^{\max}(i,j,t)$	maximum flow of the plant during stage t according to trajectory i–j (m^3/s).
$q^{\min}(i,j,t)$	minimum flow of the plant during stage t according to trajectory i–j (m^3/s).
$s(i,j,t)$	flow spilled over the spillway crest during stage t according to trajectory i–j (m^3/s)
u_{ijk}	flow released through unit k according to trajectory i–j (m^3/s)
$\bar{v}(i,j,t)$	average reservoir volume during stage t according to trajectory i–j (hm^3)
$v(j,t)$	reservoir volume at the end of stage t in state j (hm^3)
v_0	initial reservoir volume (hm^3)
v_0^a	initial reservoir volume at the beginning of the hydrological year (hm^3)
$v_0^{t_w}$	reservoir volume at the beginning of week t_w (hm^3)
v_f	final reservoir volume (hm^3)
v_f^a	final reservoir volume at the end of the hydrological year (hm^3)
$v_f^{t_w}$	target reservoir volume at the end of week t_w (hm^3)
$v_{\max}$	maximum reservoir volume (hm^3)
$v_{\min}$	minimum reservoir volume (hm^3)
v_r	relaxed final reservoir volume (hm^3)
$w(t)$	average water inflow to the reservoir during stage t (m^3/s)
x_{ik}	available flow at the beginning of interval k from state i (m^3/s)
Δv	discretization interval (hm^3)
$\pi(t)$	energy price during stage t (cent. €/kWh)

most popular optimization technique applied to solve hydro scheduling problems, whatever the time horizon or the predominant source of energy in the system [17]. The interest on dynamic programming lies mainly in the fact that it allows easily tackling nonlinear, non-concave and even discontinuous problems.

One of the main difficulties faced by most hydro scheduling models lies in the correct modeling of the head variation and its effects (hereinafter referred to as *head effects*) on the units' efficiency and operating limits (maximum and minimum flows). The power generated by a hydro plant is a nonlinear and, generally, non-concave function of the water discharge and the net head. This 3D relationship is usually represented by a family of power–discharge curves, each corresponding to a different value of the head or the volume, its shape depending, among other factors, on the type and characteristics of the plant's turbine generating units [18]. In the technical literature, reference to this mathematical model has been made through different names such as, for example, *water rate curves* [19], *characteristic curves* [20], *generation characteristic* [21] or *production function* [9], among others. In short-term studies the head effects are usually neglected, especially in the case of large reservoirs, since their variation throughout the planning time horizon is barely significant. However, in the case of smaller reservoirs, with daily or hourly regulation capability, it is really important to consider the head variation to get optimal or near-optimal realistic schedules.

It seems clear that the head effects can not be considered in linear programming models; actually, in those cases, it is usual to use a single plant/unit performance curve [10] or to model just the so-called local best efficiency points [22]. Nevertheless, the head effects have been partially considered in two mixed-integer linear programming based models [20,21]. In the first reference, each hydro unit generation is modeled by a mesh of operating points suitably selected. From these points, the remaining operating points are interpolated using an appropriate 0–1 mixed-integer linear formulation [23]; nevertheless, neither the existence of the units' minimum flow nor the dependence of the units' maximum flow with the head of the associated reservoir are considered. In turn, in the second reference an appropriate mixed-integer linear formulation is used to model the hydro plant generation by a set of piecewise linear and non-concave curves each corresponding to an interval of reservoir levels; however, minimum and maximum flows are considered constant irrespective of the reservoir water level.

The second remarkable difficulty of most hydro scheduling models comes from the discrete character of the planning decisions concerning the units' start-up and shut-down. This problem can be easily tackled by means of binary variables in mixed-integer linear models, however, in nonlinear models this approach leads to mixed-integer nonlinear models, the solution of which has not yet reached the maturity of linear or nonlinear programming methods [24]. Recently, a novel approach based on mixed integer quadratic programming was presented in [11], where both the head effects and the discrete decisions concerning the on–off behavior of the plant were successfully considered during the optimization process. Nevertheless, due to the nature of the algorithm, the plant efficiency was considered as a linear function of the head, but not of the water discharge, and the plant power generation was modeled as a quadratic function of the water discharge and the volume of the associated reservoir, what may lead to important inaccuracies in those cases where the hydro plant is composed by more than one unit [25].

Therefore, due to the inherent complexity of this problem, a great number of approaches found in the technical literature decompose it into two more simple problems: on one hand, the optimal selection of the units in operation is done (*unit commitment*); and, on the other hand, the power generated by each unit in operation is determined (*generation dispatch*). In [26], the unit commitment is solved by means of a heuristic approach and Lagrangian relaxation technique is used to solve the generation dispatch, whereas in [27], a genetic algorithm solves the unit commitment and Lagrangian relaxation is again used to solve the generation dispatch.

Dynamic programming poses some difficulties to manage the units' start-up and shut-down, unless the number of generating units is taken as a state variable [28] or as the decision stages of the problem [29]; both approaches leading to very complex